



CLASS 12 · CBSE MATHEMATICS (041)

Mathematics Solved Board Papers

Chapter-wise and year-wise CBSE board questions with complete step-by-step solutions

10 years · 2016–2026

- ✓ Part A: every question sorted chapter-wise, grouped by marks, with full solutions
- ✓ Part B: 17 complete board papers, each followed by its solutions
- ✓ Board weightage table and most-asked topics for every chapter
- ✓ Every step of every proof, integral and calculation shown the way the board awards marks
- ✓ Only the 2026-27 syllabus: questions on deleted topics are flagged

WHAT YOU GET

10 years of CBSE Maths papers. Every question solved. Sorted chapter-wise.

17

board papers

603

solved questions

10

years · 2016–26

13

chapters

23

diagrams

X Questions scattered across years



✓ All in one place, chapter-wise

X Don't know what to study first



✓ Weightage + most-asked topics

X One-line answers online



✓ Full solutions, board-style

X Old books carry deleted topics



✓ Only the 2026-27 syllabus

PART A

Chapter-wise**A**

- ✓ Weightage table
- ✓ Most-asked topics
- ✓ Every question + solution

PART B

Full papers**B**

- ✓ Exactly as set by CBSE
- ✓ Solutions after each paper
- ✓ Timed 3-hour practice

2026 · 65/1/1

2026 · 65/2/1

2025 · 65/1/1

2025 · 65/2/1

2024 · 65/1/1

2024 · 65/2/1

2023 · 65/1/1

2023 · 65/2/1

2022 · 65/1/1

2022 · 65/2/1

2020 · 65/1/1

2020 · 65/2/1

2019 · 65/1/1

2019 · 65/2/1

2018 · 65/1

2017 · 65/1

2016 · 65/1



Complete Notes · Solved PYQs

CLASS 6–12 · ALL SUBJECTS

100%
QUESTIONS
SOLVED
Made for **board toppers**

How to use this book

Part A puts every board question of a chapter together, from 1-mark MCQs to 5-mark long answers. Finish a chapter in your notes, then solve its Part A questions without looking at the solutions. Check your answer, mark the ones you got wrong, and repeat them a week later.

Each question shows where it was asked, for example **CBSE 2026 · 65/1/1 · Q13**. Use the weightage table and the most-asked topics chart at the start of each chapter to decide what to revise first.

Part B gives full papers. In the last month before the exam, sit with a clock and write one paper in 3 hours, then check it with the solutions that follow the paper.

Solutions are written the way the board awards marks: every step written, the property or formula used named at that step, and the final answer stated clearly. Write your answers the same way.

Contents

Part A · Chapter-wise questions with solutions

- | | |
|---|--------------------------------------|
| 1. Relations and Functions 26 Qs | 7. Integrals 84 Qs |
| 2. Inverse Trigonometric Functions 28 Qs | 8. Application of Integrals 27 Qs |
| 3. Matrices 36 Qs | 9. Differential Equations 49 Qs |
| 4. Determinants 47 Qs | 10. Vector Algebra 59 Qs |
| 5. Continuity and Differentiability 65 Qs | 11. Three Dimensional Geometry 51 Qs |
| 6. Application of Derivatives 45 Qs | 12. Linear Programming 33 Qs |
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Part B · Full board papers, each followed by its solutions

- | | |
|---------------------------------|----------------------------------|
| 1. CBSE 2026 · 65/1/1 solutions | 9. CBSE 2022 · 65/1/1 solutions |
| 2. CBSE 2026 · 65/2/1 solutions | 10. CBSE 2022 · 65/2/1 solutions |
| 3. CBSE 2025 · 65/1/1 solutions | 11. CBSE 2020 · 65/1/1 solutions |
| 4. CBSE 2025 · 65/2/1 solutions | 12. CBSE 2020 · 65/2/1 solutions |
| 5. CBSE 2024 · 65/1/1 solutions | 13. CBSE 2019 · 65/1/1 solutions |
| 6. CBSE 2024 · 65/2/1 solutions | 14. CBSE 2019 · 65/2/1 solutions |
| 7. CBSE 2023 · 65/1/1 solutions | 15. CBSE 2018 · 65/1 solutions |
| 8. CBSE 2023 · 65/2/1 solutions | 16. CBSE 2017 · 65/1 solutions |
| | 17. CBSE 2016 · 65/1 solutions |

603 questions in all. Questions are taken from the CBSE Class 12 Mathematics board papers and the solutions are our own, checked against the official marking schemes. Questions on topics deleted from the current syllabus (binary operations, properties of determinants, Rolle's and mean value theorems, tangents and normals, approximations, definite integral as a limit of a sum, scalar triple product, the plane, binomial distribution and others) are left out of Part A and marked in Part B.

CHAPTER 1

Relations and Functions

Board weightage 2016–2026

Year	2016	2017	2018	2019	2020	2022	2023	2024	2025	2026
Questions	–	–	1	2	5	–	3	6	4	5
Marks	–	–	6	8	11	–	12	18	12	15

Most asked topics

Equivalence relation:	1
$f(x) = (3x+2)/(5x-3)$ is one-one and onto	1
Type of a relation on $\{1, 2, 3\}$	1
Is $(x-2)/(x-3)$ onto R ?	1
Injectivity of $f(x, y) = (2y, 3x)$	1
$f(x) = 3x - 5$ on Z ($A \rightarrow R$)	1
Five relations on $\{1, 2, 3\}$ (case)	1
Relation "m is a multiple of n" on N	1

26 questions from 13 papers. Repeated questions are shown once with every year they were asked.

1 mark: MCQ and Assertion–Reason

CBSE 2026 · 65/2/1 · Q1

1 mark

A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (2, 2), (1, 2)\}$ is

- (A) Reflexive only (B) Reflexive and Transitive
(C) Symmetric and Transitive (D) Transitive only

Solution

Answer: (D) Transitive only

Not reflexive: $(3, 3) \notin R$. Not symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$. Transitive: the only chains are $(1, 1), (1, 2) \Rightarrow (1, 2) \in R$ and $(1, 2), (2, 2) \Rightarrow (1, 2) \in R$.

CBSE 2025 · 65/1/1 · Q19

1 mark

Assertion (A) : Let Z be the set of integers. A function $f : Z \rightarrow Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

Solution

Answer: (D)

f is one-one, but not onto $\mathbb{Z}$: e.g. $3x - 5 = 0$ gives $x = \frac{5}{3} \notin \mathbb{Z}$, so 0 has no pre-image. A is false; R is the definition of a bijection (true).

CBSE 2024 · 65/1/1 · Q1

1 mark

A function $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ (where $\mathbb{R}_+$ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

- (A) one-one but not onto (B) onto but not one-one
(C) both one-one and onto (D) neither one-one nor onto

Solution**Answer: (A)**

$4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$: one-one. For $x \geq 0$, $f(x) \geq 3$, so values below 3 (e.g. 0) have no pre-image: not onto $\mathbb{R}$.

CBSE 2024 · 65/2/1 · Q2

1 mark

Let $f : \mathbb{R}_+ \rightarrow [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where $\mathbb{R}_+$ is the set of all non-negative real numbers. Then, f is :

- (A) one-one (B) onto
(C) bijective (D) neither one-one nor onto

Solution**Answer: (C) bijective**

$f(x) = (3x + 1)^2 - 6$. For $x \geq 0$, $3x + 1 \geq 1$ and f is strictly increasing (one-one). Its values run from $f(0) = -5$ upward without bound, covering $[-5, \infty)$ (onto): for $y \geq -5$, $x = \frac{\sqrt{y+6}-1}{3} \geq 0$.

CBSE 2020 · 65/1/1 · Q11

1 mark

A relation R in a set A is called _____, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

Solution**Symmetric**

CBSE 2020 · 65/2/1 · Q1

1 mark

The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is

- (A) symmetric and transitive, but not reflexive (B) reflexive and symmetric, but not transitive
(C) symmetric, but neither reflexive nor transitive (D) an equivalence relation

Solution**Answer: (C)**

$(2, 2), (3, 3) \notin R$: not reflexive. $(1, 2)$ and $(2, 1)$ both in R : symmetric. $(2, 1), (1, 2) \in R$ but $(2, 2) \notin R$: not transitive.

CBSE 2020 · 65/2/1 · Q5

1 mark

Let $A = \{1, 3, 5\}$. Then the number of equivalence relations in A containing $(1, 3)$ is

- (A) 1 (B) 2
(C) 3 (D) 4

Solution**Answer: (B) 2**

1 and 3 must be in the same class: either $\{1, 3\}$, $\{5\}$ or $\{1, 3, 5\}$ (the universal relation).

2 marks: very short answer

CBSE 2026 · 65/2/1 · Q21(a)

2 marks

Check whether $f : R - \{3\} \rightarrow R$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not.

Solution

$$\text{Let } y = \frac{x-2}{x-3}: xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}.$$

This is not defined for $y = 1$, i.e. no x gives $f(x) = 1$ (indeed $\frac{x-2}{x-3} = 1$ gives $-2 = -3$, impossible).

Range $= R - \{1\} \neq R$ (codomain), so **f is not onto**.

CBSE 2026 · 65/2/1 · Q21(b)

2 marks

Check whether $f : Z \times Z \rightarrow Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.

Solution

$$\text{Let } f(x_1, y_1) = f(x_2, y_2): (2y_1, 3x_1) = (2y_2, 3x_2) \Rightarrow 2y_1 = 2y_2 \text{ and } 3x_1 = 3x_2 \Rightarrow y_1 = y_2, x_1 = x_2.$$

So $(x_1, y_1) = (x_2, y_2)$: **f is injective**.

CBSE 2023 · 65/1/1 · Q21(a)

2 marks

A function $f : A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B .

Solution

f is onto, so B equals the range: $f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$.

$B = \{2, 4, 6, 8\}$

3 marks: short answer

CBSE 2025 · 65/2/1 · Q27

3 marks

Let R be a relation defined over N , where N is set of natural numbers, defined as " mRn if and only if m is a multiple of n , $m, n \in N$." Find whether R is reflexive, symmetric and transitive or not.

Solution

Reflexive: every natural number is a multiple of itself ($m = 1 \cdot m$), so mRm . R is reflexive.

Not symmetric: $8R2$ (8 is a multiple of 2) but 2 is not a multiple of 8.

Transitive: if mRn and nRp then $m = kn$ and $n = lp$ for natural k, l , so $m = (kl)p$: mRp .

R is reflexive and transitive but not symmetric.

CBSE 2024 · 65/1/1 · Q26(a)

3 marks

A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

Solution

Reflexive: $|x^2 - x^2| = 0 < 8$ for every x , so $(x, x) \in R$.

Symmetric: $|y^2 - x^2| = |x^2 - y^2|$, so $(x, y) \in R \Rightarrow (y, x) \in R$.

Not transitive: $(1, 2) \in R$ ($|1 - 4| = 3$) and $(2, 3) \in R$ ($|4 - 9| = 5$), but $(1, 3) \notin R$ ($|1 - 9| = 8$, not < 8).

R is reflexive and symmetric but not transitive.

CBSE 2024 · 65/1/1 · Q26(b)

3 marks

A function f is defined from $R \rightarrow R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not.

Solution

$$a + b = 1, 2a + b = 3 \Rightarrow a = 2, b = -1: f(x) = 2x - 1.$$

$$\text{One-one: } 2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2.$$

$$\text{Onto: for any } y \in R, x = \frac{y+1}{2} \in R \text{ and } f(x) = y.$$

f is one-one and onto.

4 marks: case-based / source-based

CBSE 2025 · 65/1/1 · Q37

4 marks

A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$: $R_1 = \{(2, 3), (3, 2)\}$; $R_2 = \{(1, 2), (1, 3), (3, 2)\}$; $R_3 = \{(1, 2), (2, 1), (1, 1)\}$; $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$; $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$.

- (i) Identify the relation which is reflexive, transitive but not symmetric. (1)
 (ii) Identify the relation which is reflexive and symmetric but not transitive. (1)
 (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. (2)
 OR
 (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation? (2)

Solution

- (i) R_4 : contains (1, 1), (2, 2), (3, 3) (reflexive); (1, 2) without (2, 1) (not symmetric); every chain gives (1, 2), which is present (transitive).
 (ii) R_5 : reflexive and symmetric, but (1, 2), (2, 3) $\in R_5$ while (1, 3) $\notin R_5$, so not transitive.
 (iii) (a) R_1 and R_3 . R_1 : symmetric; not reflexive; (2, 3), (3, 2) but (2, 2) $\notin R_1$ — not transitive. R_3 : symmetric; (2, 2) $\notin R_3$ — not reflexive; (2, 1), (1, 2) but (2, 2) $\notin R_3$ — not transitive.
 OR (iii) (b) Reflexivity needs (1, 1), (2, 2), (3, 3); symmetry needs (2, 1), (3, 1), (2, 3). With these all nine pairs are present ($A \times A$), which is transitive.
 Pairs to add: **(1, 1), (2, 2), (3, 3), (2, 1), (3, 1), (2, 3)**

CBSE 2025 · 65/2/1 · Q36

4 marks

A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

- (i) How many relations can be there from S to J ? (1)
 (ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$. Check if it is bijective. (1)
 (iii) (a) How many one-one functions can be there from set S to set J ? (2)
 OR
 (iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S . Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. (2)

Solution

- (i) $S \times J$ has $4 \times 3 = 12$ elements; relations = subsets: $2^{12} = 4096$
 (ii) $f(S_2) = f(S_3) = J_2$, so f is not one-one: **not bijective** (it is onto).
 (iii) (a) A one-one function from a 4-element set into a 3-element set is impossible (two speakers must share a judge): **0**
 OR (iii) (b) Add **(S₁, S₁), (S₂, S₂), (S₃, S₃), (S₄, S₄)**. R_1 then is reflexive, and still not symmetric since $(S_1, S_2) \in R_1$ but $(S_2, S_1) \notin R_1$.

CBSE 2020 · 65/1/1 · Q27

4 marks

Let N be the set of natural numbers and R be the relation on $N \times N$ defined by $(a, b) R (c, d)$ iff $ad = bc$ for all $a, b, c, d \in N$. Show that R is an equivalence relation.

Solution

Reflexive: $ab = ba$, so $(a, b) R (a, b)$.

Symmetric: $ad = bc \Rightarrow cb = da$, so $(c, d) R (a, b)$.

Transitive: $(a, b) R (c, d)$ and $(c, d) R (e, f)$ give $ad = bc$ and $cf = de$. Multiply: $adc f = bcde \Rightarrow af = be$ (cancelling $cd \neq 0$), so $(a, b) R (e, f)$.

Hence R is an equivalence relation.

CBSE 2020 · 65/2/1 · Q27(a)

4 marks

Check whether the relation R in the set N of natural numbers given by $R = \{(a, b) : a \text{ is divisor of } b\}$ is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

Solution

Reflexive: every a divides itself.

Not symmetric: 2 divides 4 but 4 does not divide 2.

Transitive: $a \mid b$ and $b \mid c \Rightarrow b = ka, c = lb = (kl)a \Rightarrow a \mid c$.

Since it is not symmetric, **R is not an equivalence relation.**

CBSE 2019 · 65/1/1 · Q13(a)

4 marks

Show that the relation R on R defined as $R = \{(a, b) : a \leq b\}$, is reflexive, and transitive but not symmetric.

Solution

Reflexive: $a \leq a$ for every real a , so $(a, a) \in R$.

Transitive: $a \leq b$ and $b \leq c \Rightarrow a \leq c$.

Not symmetric: $(1, 2) \in R$ since $1 \leq 2$, but $(2, 1) \notin R$.

CBSE 2019 · 65/2/1 · Q13(a)

4 marks

Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Solution

$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$.

Not reflexive: $(1, 1) \notin R$. **Not symmetric:** $(1, 2) \in R$, $(2, 1) \notin R$. **Not transitive:** $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R$.

5 marks: long answer

CBSE 2026 · 65/1/1 · Q32(a)

5 marks

A relation R is defined on Z , the set of integers, as $R = \{(x, y) : |x - y| \text{ is divisible by a prime number 'p', } x, y \in Z\}$; check whether R is an equivalence relation or not.

Solution

Reflexive: for any $x \in Z$, $|x - x| = 0 = 0 \cdot p$ is divisible by p , so $(x, x) \in R$.

Symmetric: if $(x, y) \in R$, then $|x - y| = kp$ for some integer k . Since $|y - x| = |x - y| = kp$, $(y, x) \in R$.

Transitive: if $(x, y) \in R$ and $(y, z) \in R$, then $x - y = mp$ and $y - z = np$ for integers m, n (signs absorbed in m, n). Adding: $x - z = (m + n)p$, so $|x - z|$ is divisible by p and $(x, z) \in R$.

R is reflexive, symmetric and transitive, so **R is an equivalence relation.**

CBSE 2026 · 65/1/1 · Q32(b)

5 marks

A function $f : R - \left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x + 2}{5x - 3}$. Show that f is one-one and onto.

Solution

One-one: let $f(x_1) = f(x_2)$: $\frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3}$

$$(3x_1 + 2)(5x_2 - 3) = (3x_2 + 2)(5x_1 - 3)$$

$15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 - 9x_2 + 10x_1 - 6 \Rightarrow -19x_1 = -19x_2 \Rightarrow x_1 = x_2$. So f is one-one.

Onto: let $y \in R - \left\{\frac{3}{5}\right\}$ and $f(x) = y$: $3x + 2 = 5xy - 3y \Rightarrow x(5y - 3) = 3y + 2 \Rightarrow x = \frac{3y + 2}{5y - 3}$, which is a real number since $y \neq \frac{3}{5}$.

Also $x \neq \frac{3}{5}$: if $\frac{3y + 2}{5y - 3} = \frac{3}{5}$ then $15y + 10 = 15y - 9$, impossible. So $x \in R - \left\{\frac{3}{5}\right\}$ and $f(x) = y$; f is onto.

Hence f is one-one and onto.

CBSE 2024 · 65/2/1 · Q32(a)

5 marks

Show that a function $f : R \rightarrow R$ defined by $f(x) = \frac{2x}{1 + x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \rightarrow A$ becomes an onto function.

Solution

Not one-one: $f(2) = \frac{4}{5}$ and $f\left(\frac{1}{2}\right) = \frac{1}{5/4} = \frac{4}{5}$, but $2 \neq \frac{1}{2}$.

Not onto: let $y = \frac{2x}{1 + x^2} \Rightarrow yx^2 - 2x + y = 0$. For real x ($y \neq 0$), discriminant $4 - 4y^2 \geq 0 \Rightarrow -1 \leq y \leq 1$. So e.g. $y = 2$ has no pre-image; f is not onto R .

Range = $[-1, 1]$ ($y = 0$ from $x = 0$; $y = \pm 1$ from $x = \pm 1$). So f is onto for $A = [-1, 1]$.

CBSE 2024 · 65/2/1 · Q32(b)

5 marks

A relation R is defined on $N \times N$ (where N is the set of natural numbers) as : $(a, b) R (c, d) \Leftrightarrow a - c = b - d$. Show that R is an equivalence relation.

Solution

Note $a - c = b - d \Leftrightarrow a - b = c - d$.

Reflexive: $a - a = b - b = 0$, so $(a, b) R (a, b)$.

Symmetric: if $a - c = b - d$ then $c - a = d - b$, so $(c, d) R (a, b)$.

Transitive: $(a, b) R (c, d)$ and $(c, d) R (e, f)$ give $a - b = c - d$ and $c - d = e - f$, so $a - b = e - f$, i.e. $a - e = b - f$: $(a, b) R (e, f)$.

Hence R is an equivalence relation.

CBSE 2023 · 65/1/1 · Q32

5 marks

A relation R is defined on a set of real numbers R as $R = \{(x, y) : x \cdot y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not.

Solution

Not reflexive: $1 \cdot 1 = 1$ is rational, so $(1, 1) \notin R$.

Symmetric: $xy = yx$, so if xy is irrational, so is yx : $(x, y) \in R \Rightarrow (y, x) \in R$.

Not transitive: $(1, \sqrt{2}) \in R$ (product $\sqrt{2}$) and $(\sqrt{2}, 1) \in R$, but $(1, 1) \notin R$.

R is symmetric but neither reflexive nor transitive.

CBSE 2023 · 65/2/1 · Q35

5 marks

A function $f: [-4, 4] \rightarrow [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

Solution

Onto: for any $y \in [0, 4]$, take $x = \sqrt{16 - y^2} \in [0, 4] \subset [-4, 4]$; then $f(x) = \sqrt{16 - (16 - y^2)} = y$.

Not one-one: $f(-4) = f(4) = 0$ (in general $f(-x) = f(x)$).

$f(a) = \sqrt{7} \Rightarrow 16 - a^2 = 7 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$

6 marks: long answer

CBSE 2018 · 65/1 · Q24(a)

6 marks

Let $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class $[2]$.

Solution

Reflexive: $|a - a| = 0$ is divisible by 4. **Symmetric:** $|b - a| = |a - b|$. **Transitive:** $a - b = 4m$, $b - c = 4n \Rightarrow a - c = 4(m + n)$.

So R is an equivalence relation.

Elements related to 1: $|1 - b|$ divisible by 4: **{1, 5, 9}**. Equivalence class **[2] = {2, 6, 10}**.

CBSE BOARD PAPER

2026 · Q.P. Code 65111

Time: 3 hours

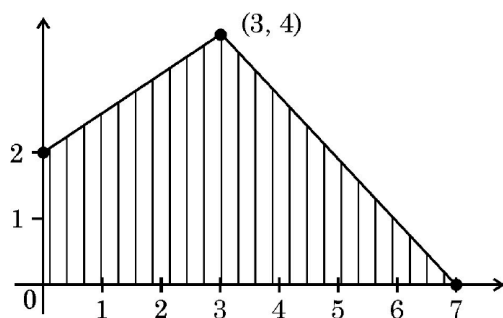
Maximum marks: 80

Section A · Multiple choice and Assertion–Reason (1 mark each)

1. If $2 \cos^{-1} x = y$, then 1
 (A) $0 \leq y \leq \pi$ (B) $-\pi \leq y \leq \pi$
 (C) $0 \leq y \leq 2\pi$ (D) $-\pi \leq y \leq 0$
2. Which of the following cannot be the order of a row-matrix? 1
 (A) 2×1 (B) 1×2
 (C) 1×1 (D) $1 \times n$
3. Which of the following properties is/are true for two matrices of suitable orders? (i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$ (iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar) 1
 (A) (i) only (B) (i), (ii) and (iii)
 (C) (i) and (ii) (D) (i) and (iv)
4. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then 1
 (A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$
 (C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$
5. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is 1
 (A) 0 (B) $\frac{\pi}{4}$
 (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$
6. If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric? 1
 (A) AB (B) AB + BA
 (C) $(A + B)^2$ (D) A - B
7. The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is 1
 (A) -16 (B) -9
 (C) 0 (D) 16
8. If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log |b^2 + c^2x^2| + K$, then the value of A is 1
 (A) 3a (B) $\frac{3a}{2b^2}$
 (C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$

9. The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx$ is 1
 (A) 0 (B) $\log 2$
 (C) $2 \log 2$ (D) $\frac{1}{2} \log 2$
10. The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by 1
 (A) 0 (B) $\frac{1}{3}$
 (C) $\frac{2}{3}$ (D) $\frac{4}{3}$
11. The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is 1
 (A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$
 (C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$
12. The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are 1
 (A) 0, 1 (B) 1, 1
 (C) 2, 1 (D) 1, not defined
13. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is 1
 (A) 0 (B) 1
 (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$
14. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is 1
 (A) 8 (B) -8
 (C) 2 (D) $\frac{5}{2}$
15. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $|\vec{a} \cdot \vec{b}|$ 1
 (A) $6\sqrt{3}$ (B) $8\sqrt{3}$
 (C) $12\sqrt{3}$ (D) $3\sqrt{12}$
16. The length of perpendicular drawn from point (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is 1
 (A) 2 (B) 5
 (C) $\sqrt{74}$ (D) $\sqrt{78}$

17. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below :



The maximum value of Z – minimum value of Z is

- (A) 8 (B) 29
(C) 35 (D) 43
18. The degree of an objective function of a linear programming problem is
- (A) 0 (B) 1
(C) 2 (D) Any natural number

19. Assertion (A) : In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$.

Reason (R) : For any two events A and B, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

20. Assertion (A) : Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$.

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

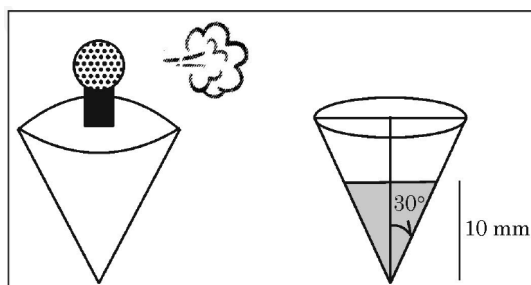
Section B · Very short answer (2 marks each)

21. (a) Check whether function $f(x)$ defined as $f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not ?

OR

- (b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

22. A room freshner bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of $1 \text{ mm}^3/\text{min}$. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm , if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$.



23. Find the vector of magnitude 14 in the direction of $\vec{QP}$, where P and Q are the points $(1, 3, 2)$ and $(-1, 0, 8)$ respectively.
24. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.
25. (a) Simplify : $\tan^{-1} \left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right), 0 < x < \frac{\pi}{4}$.
- OR
- (b) Evaluate : $\tan \left(\sin^{-1} 1 - \cos^{-1} \left(-\frac{1}{2} \right) \right)$

Section C · Short answer (3 marks each)

26. Evaluate : $\int_0^1 x \tan^{-1} x \, dx$.
27. (a) Find $\int \sqrt{\frac{x+2}{x-2}} \, dx$
- OR
- (b) Find : $\int \frac{x^2}{(x^2+9)(x^2+16)} \, dx$
28. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x| \, dx$, then show that $I_1 - 4I_2 = 0$.
29. (a) Find the general solution of the following differential equation : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$
- OR
- (b) Find the particular solution of the differential equation $xy \frac{dy}{dx} = (x+2)(y+2)$, given that $y(1) = -1$.

30. Solve the following linear programming problem graphically : Minimize $Z = 13x - 15y$ subject to constraints $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$ 3
31. (a) Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball. 3
- OR**
- (b) The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$. 3

Section D · Long answer (5 marks each)

32. (a) A relation R is defined on Z, the set of integers, as $R = \{(x, y) : |x - y| \text{ is divisible by a prime number 'p', } x, y \in Z\}$; check whether R is an equivalence relation or not. 5
- OR**
- (b) A function $f : R - \left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x + 2}{5x - 3}$. Show that f is one-one and onto. 5
33. (a) If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations 5
 $-2y + z = 7$, $2x - y - z = 8$, $x - 2y = 10$
- OR**
- (b) If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$. 5
34. If $x = \cos t$, $y = \cos mt$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$. 5
35. Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting. 5

Section E · Case-based (4 marks each)

36. An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries. The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x . 4
- (i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? (1)
(ii) If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x . (1)
(iii) (a) Find the value of x for which $R(x)$ is maximum. (2)
OR
(iii) (b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing. (2)
37. In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each. A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000. Rohan bought one ticket. 4
- (i) What are the possible amounts, the person can win ? (1)
(ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? (2)
OR
(ii) (b) What is the probability that the person does not win any amount ? (2)
(iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot. (1)
38. Roundabouts are often made on busy roads to ease the traffic and avoid red lights. One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$. There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$. 4
- (i) Represent the given equations C_1 and C_2 with the help of a diagram. (1)
(ii) Express y as a function of x , ($y = f(x)$), for both C_1 and C_2 . (1)
(iii) (a) Using integration find the area of region covered by the roundabout. (2)
OR
(iii) (b) Using integration, find the area of region covered by circular pond. (2)

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Q1 Answer: (C)

The principal value range of $\cos^{-1} x$ is $[0, \pi]$, so $0 \leq \cos^{-1} x \leq \pi$ gives $0 \leq 2 \cos^{-1} x \leq 2\pi$, i.e. $0 \leq y \leq 2\pi$.

Q2 Answer: (A) 2×1

A row matrix has exactly one row, so its order is $1 \times n$. A 2×1 matrix has two rows (it is a column matrix).

Q3 Answer: (D)

$(A + B)' = A' + B'$ is true. $(A - B)' = A' - B'$, not $B' - A'$, so (ii) is false. $(AB)' = B'A'$ (order reversed), so (iii) is false. $(kAB)' = k(AB)' = kB'A'$, so (iv) is true.

Q4 Answer: (B)

Δ_1 is a diagonal determinant: $\Delta_1 = 1 \times 2 \times 3 = 6$.

Expanding Δ_2 along R_1 : $\Delta_2 = 0 - 2 \begin{vmatrix} 1 & 0 \\ 0 & 6 \end{vmatrix} + 0 = -2(6) = -12$.

So $\Delta_2 = -12 = -2\Delta_1$.

Q5 **Answer: (B)** $\frac{\pi}{4}$

$\cos x \sin x - (-\cos x) \sin x = 2 \sin x \cos x = \sin 2x = 1$, so $2x = \frac{\pi}{2}$, $x = \frac{\pi}{4}$.

Q6 **Answer: (D) A – B**

$A' = -A$, $B' = -B$. Then $(A - B)' = A' - B' = -A + B = -(A - B)$, so $A - B$ is skew symmetric.

(For (B): $(AB + BA)' = B'A' + A'B' = BA + AB$, which is symmetric.)

Q7 **Answer: (A) -16**

$f'(x) = 3x^2 - 12 = 0 \Rightarrow x = 2$ (in $[0, 3]$).

$f(0) = 0$, $f(2) = 8 - 24 = -16$, $f(3) = 27 - 36 = -9$. Least value = -16 .

Q8 **Answer: (D)**

Put $t = b^2 + c^2x^2$, $dt = 2c^2x dx$: $\int \frac{3ax}{t} \cdot \frac{dt}{2c^2x} = \frac{3a}{2c^2} \log |t| + K$. So $A = \frac{3a}{2c^2}$.

Q9 **Answer: (A) 0**

$f(-x) = \frac{-x^3}{x^2 + 2|x| + 1} = -f(x)$, so the integrand is odd, and $\int_{-a}^a f(x) dx = 0$ for an odd function.

Q10 **Answer: (C)** $\frac{2}{3}$

$y = x^2$ for $x \geq 0$ and $y = -x^2$ for $x < 0$; the two parts are symmetric about the origin (one above, one below the x-axis). Area = $2 \int_0^1 x^2 dx = 2 \times \frac{1}{3} = \frac{2}{3}$.

Q11 **Answer: (C)**

Divide by R: $\frac{dx}{dy} + \frac{P}{R}x = \frac{Q}{R}$, linear in x. IF = $e^{\int \frac{P}{R} dy}$.

Q12 **Answer: (B) 1, 1**

$\frac{d}{dx}(e^y) = e^y \frac{dy}{dx} = 0$. The highest derivative is $\frac{dy}{dx}$ (order 1), occurring to the first power (degree 1).

Q13 **Answer: (C)** $\frac{5}{2}$

Perpendicular: dot product = 0: $1(2) + 2(-p) + 3(1) = 0 \Rightarrow 5 - 2p = 0 \Rightarrow p = \frac{5}{2}$.

Q14 **Answer: (A) 8**

$$\vec{AB} = 3\hat{i} + (m+1)\hat{j} + 3\hat{k}, \vec{AC} = 4\hat{i} + 12\hat{j} + 4\hat{k}. \text{ Collinear} \Rightarrow \text{components proportional:}$$

$$\frac{3}{4} = \frac{m+1}{12} = \frac{3}{4} \Rightarrow m+1 = 9 \Rightarrow m = 8.$$

Q15 **Answer: (C)** $12\sqrt{3}$

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2: 144 + (\vec{a} \cdot \vec{b})^2 = 64 \times 9 = 576, \text{ so } (\vec{a} \cdot \vec{b})^2 = 432, |\vec{a} \cdot \vec{b}| = \sqrt{432} = 12\sqrt{3}.$$

Q16 **Answer: (C)** $\sqrt{74}$

The line passes through the origin with direction (1, 0, 0): it is the x-axis. The foot of the perpendicular from (2, 5, 7) is (2, 0, 0), so the length = $\sqrt{5^2 + 7^2} = \sqrt{74}$.

Q17 **Answer: (D)** 43

Corner points: (0, 0), (0, 2), (3, 4), (7, 0).

Z: (0, 0) $\rightarrow$ 0; (0, 2) $\rightarrow$ 14; (3, 4) $\rightarrow$ 15 + 28 = 43; (7, 0) $\rightarrow$ 35. Max = 43, min = 0, difference = 43.

Q18 **Answer: (B)** 1

The objective function $Z = ax + by$ is linear, so its degree is 1.

Q19 **Answer: (C)**

Odd outcomes {1, 3, 5}; primes among them {3, 5}. $P(\text{prime}|\text{odd}) = \frac{2/6}{3/6} = \frac{2}{3}$, so A is true.

R is false: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Q20 **Answer: (D)**

First line: $\frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r}$, direction (p, 1, r); second: (p', 1, r'). Perpendicular $\Leftrightarrow pp' + 1 + rr' = 0$, i.e. $pp' + rr' = -1$, not 1. So A is false; R is true.

Q21(a) For $x < 3$, $|x-3| = -(x-3)$, so $f(x) = \frac{-(x-3)}{2(x-3)} = -\frac{1}{2}$.

$$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = -\frac{1}{2}.$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} \frac{x-6}{6} = \frac{3-6}{6} = -\frac{1}{2}; f(3) = \frac{3-6}{6} = -\frac{1}{2}.$$

LHL = RHL = f(3), so **f is continuous at x = 3**.

Q21(b) Differentiate w.r.t. x: $\sqrt{3} \left(2x + 2y \frac{dy}{dx} \right) = 4y + 4x \frac{dy}{dx}$

$$\frac{dy}{dx} (2\sqrt{3}y - 4x) = 4y - 2\sqrt{3}x \Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$$

$$\text{At } \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right): \text{numerator} = \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}; \text{denominator} = \frac{3}{2} - 1 = \frac{1}{2}.$$

$$\frac{dy}{dx} = \sqrt{3}$$

Q22

Let the level be h and the radius of the surface r . Semi-vertical angle $\frac{\pi}{6}$: $r = h \tan \frac{\pi}{6} = \frac{h}{\sqrt{3}}$.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot \frac{h^2}{3} \cdot h = \frac{\pi h^3}{9}$$

$$\frac{dV}{dt} = \frac{\pi h^2}{3} \frac{dh}{dt} = -1 \text{ (volume decreases)}$$

$$\text{At } h = 10: \frac{100\pi}{3} \frac{dh}{dt} = -1 \Rightarrow \frac{dh}{dt} = -\frac{3}{100\pi}$$

The level is dropping at $\frac{3}{100\pi}$ mm/min.

Q23

$$\vec{QP} = \vec{OP} - \vec{OQ} = (1+1)\hat{i} + (3-0)\hat{j} + (2-8)\hat{k} = 2\hat{i} + 3\hat{j} - 6\hat{k}$$

$$|\vec{QP}| = \sqrt{4+9+36} = 7$$

$$\text{Required vector} = 14 \times \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7} = 4\hat{i} + 6\hat{j} - 12\hat{k}$$

Q24

Diagonals: $\vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{a} - \vec{b} = 2\hat{i} - 2\hat{j}$ (or $\vec{b} - \vec{a}$).

$$|\vec{a} + \vec{b}| = \sqrt{16+4+16} = 6; |\vec{a} - \vec{b}| = \sqrt{4+4+0} = 2\sqrt{2}$$

Q25(a) Divide numerator and denominator by $\cos 2x$:

$$\tan^{-1} \left(\frac{1 - \tan 2x}{1 + \tan 2x} \right) = \tan^{-1} \left(\tan \left(\frac{\pi}{4} - 2x \right) \right)$$

For $0 < x < \frac{\pi}{4}$, $-\frac{\pi}{4} < \frac{\pi}{4} - 2x < \frac{\pi}{4}$, which lies in the principal range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

So the expression = $\frac{\pi}{4} - 2x$

Q25(b)

$$\sin^{-1} 1 = \frac{\pi}{2}; \cos^{-1} \left(-\frac{1}{2} \right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\tan \left(\frac{\pi}{2} - \frac{2\pi}{3} \right) = \tan \left(-\frac{\pi}{6} \right) = -\frac{1}{\sqrt{3}}$$

Q26

By parts (first $\tan^{-1} x$, second x):

$$\begin{aligned} I &= \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx \\ &= \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{2} \int_0^1 \left(1 - \frac{1}{1+x^2} \right) dx = \frac{\pi}{8} - \frac{1}{2} \left[x - \tan^{-1} x \right]_0^1 \\ &= \frac{\pi}{8} - \frac{1}{2} \left(1 - \frac{\pi}{4} \right) = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2} \end{aligned}$$

Q27(a)

Multiply inside by $\frac{x+2}{x+2}$: $\sqrt{\frac{x+2}{x-2}} = \frac{x+2}{\sqrt{x^2-4}}$

$$I = \int \frac{x}{\sqrt{x^2-4}} dx + 2 \int \frac{dx}{\sqrt{x^2-4}}$$

First: put $x^2 - 4 = t$, $2x dx = dt$: $\int \frac{dt}{2\sqrt{t}} = \sqrt{t} = \sqrt{x^2-4}$.

Second: standard form $\int \frac{dx}{\sqrt{x^2-a^2}} = \log |x + \sqrt{x^2-a^2}|$.

$$I = \sqrt{x^2 - 4} + 2 \log |x + \sqrt{x^2 - 4}| + C$$

Q27(b)

Put $x^2 = t$ (only for the partial fractions): $\frac{t}{(t+9)(t+16)} = \frac{A}{t+9} + \frac{B}{t+16}$

$$t = A(t+16) + B(t+9). t = -9: -9 = 7A \Rightarrow A = -\frac{9}{7}; t = -16: -16 = -7B \Rightarrow$$

$$B = \frac{16}{7}.$$

$$I = -\frac{9}{7} \int \frac{dx}{x^2+9} + \frac{16}{7} \int \frac{dx}{x^2+16} = -\frac{9}{7} \cdot \frac{1}{3} \tan^{-1} \frac{x}{3} + \frac{16}{7} \cdot \frac{1}{4} \tan^{-1} \frac{x}{4}$$

$$I = \frac{4}{7} \tan^{-1} \frac{x}{4} - \frac{3}{7} \tan^{-1} \frac{x}{3} + C$$

Q28

$$1 + \cos 2x = 2 \cos^2 x: I_1 = \frac{1}{2} \int_{-\pi/4}^{\pi/4} \sec^2 x \, dx = \frac{1}{2} \left[\tan x \right]_{-\pi/4}^{\pi/4} = \frac{1}{2} (1 - (-1)) = 1$$

$$|x| \text{ is even: } I_2 = 2 \int_0^{1/2} x \, dx = 2 \left[\frac{x^2}{2} \right]_0^{1/2} = \frac{1}{4}$$

$$I_1 - 4I_2 = 1 - 4 \times \frac{1}{4} = 0. \text{ Hence proved.}$$

Q29(a)

$$\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2 - \text{homogeneous. Put } y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx}:$$

$$v + x \frac{dv}{dx} = 1 + v + v^2 \Rightarrow x \frac{dv}{dx} = 1 + v^2$$

$$\text{Separate: } \frac{dv}{1+v^2} = \frac{dx}{x} \Rightarrow \tan^{-1} v = \log |x| + C$$

$$\text{General solution: } \tan^{-1} \frac{y}{x} = \log |x| + C$$

Q29(b)

$$\text{Separate: } \frac{y}{y+2} dy = \frac{x+2}{x} dx \Rightarrow \left(1 - \frac{2}{y+2}\right) dy = \left(1 + \frac{2}{x}\right) dx$$

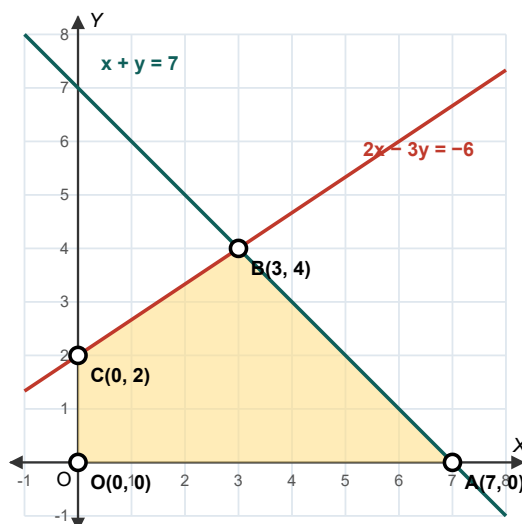
$$\text{Integrate: } y - 2 \log |y+2| = x + 2 \log |x| + C$$

$$y(1) = -1: -1 - 2 \log 1 = 1 + 2 \log 1 + C \Rightarrow C = -2$$

$$\text{Particular solution: } y - 2 \log |y+2| = x + 2 \log |x| - 2$$

Q30

Draw $x + y = 7$ (through (7, 0), (0, 7)) and $2x - 3y = -6$ (through (0, 2), (-3, 0)). The origin satisfies both inequalities, so the feasible region is on the origin side of both lines, in the first quadrant (shaded).



Corner points: $O(0, 0)$, $A(7, 0)$, $B(3, 4)$ [solving $x + y = 7$ and $2x - 3y = -6$], $C(0, 2)$.

Corner point	$Z = 13x - 15y$
$O(0, 0)$	0
$A(7, 0)$	91
$B(3, 4)$	$39 - 60 = -21$
$C(0, 2)$	-30

The region is bounded, so the minimum value is $Z = -30$ at $x = 0$, $y = 2$.

Q31(a) E_1 : bag I chosen (die shows 1 or 2): $P(E_1) = \frac{2}{6} = \frac{1}{3}$; E_2 : bag II: $P(E_2) = \frac{2}{3}$.

$$P(R|E_1) = \frac{3}{7}; P(R|E_2) = \frac{8}{14} = \frac{4}{7}.$$

$$\text{By total probability: } P(R) = \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7} = \frac{1}{7} + \frac{8}{21} = \frac{11}{21}$$

Q31(b) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = a \dots (1)$

$$P(\text{exactly one}) = P(X) + P(Y) - 2P(X \cap Y) = b \dots (2)$$

$$(1) - (2): P(X \cap Y) = a - b. \text{ From (1): } P(X) + P(Y) = a + (a - b) = 2a - b.$$

$$P(X') + P(Y') = [1 - P(X)] + [1 - P(Y)] = 2 - (2a - b) = 2 - 2a + b. \text{ Hence proved.}$$

Q32(a) **Reflexive:** for any $x \in Z$, $|x - x| = 0 = 0 \cdot p$ is divisible by p , so $(x, x) \in R$.

Symmetric: if $(x, y) \in R$, then $|x - y| = kp$ for some integer k . Since $|y - x| = |x - y| = kp$, $(y, x) \in R$.

Transitive: if $(x, y) \in R$ and $(y, z) \in R$, then $x - y = mp$ and $y - z = np$ for integers m, n (signs absorbed in m, n). Adding: $x - z = (m + n)p$, so $|x - z|$ is divisible by p and $(x, z) \in R$.

R is reflexive, symmetric and transitive, so **R is an equivalence relation.**

Q32(b) **One-one:** let $f(x_1) = f(x_2)$: $\frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3}$

$$(3x_1 + 2)(5x_2 - 3) = (3x_2 + 2)(5x_1 - 3)$$

$$15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 - 9x_2 + 10x_1 - 6 \Rightarrow -19x_1 = -19x_2 \Rightarrow x_1 = x_2.$$

So f is one-one.

Onto: let $y \in R - \left\{\frac{3}{5}\right\}$ and $f(x) = y: 3x + 2 = 5xy - 3y \Rightarrow x(5y - 3) = 3y + 2 \Rightarrow x = \frac{3y + 2}{5y - 3}$, which is a real number since $y \neq \frac{3}{5}$.

Also $x \neq \frac{3}{5}$: if $\frac{3y + 2}{5y - 3} = \frac{3}{5}$ then $15y + 10 = 15y - 9$, impossible. So $x \in R - \left\{\frac{3}{5}\right\}$ and $f(x) = y$; f is onto.

Hence f is one-one and onto.

Q33(a) $|A| = 0(0 - 2) - 2(0 + 2) + 1(2 + 1) = -4 + 3 = -1 \neq 0$, so A^{-1} exists.

Cofactors: $C_{11} = -2, C_{12} = -2, C_{13} = 3; C_{21} = -1, C_{22} = -1, C_{23} = 2; C_{31} = -3, C_{32} = -2, C_{33} = 4$.

$$\text{adj} A = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}, A^{-1} = \frac{1}{|A|} \text{adj} A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$$

The system is $\begin{bmatrix} 0 & -2 & 1 \\ 2 & -1 & -1 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$; the coefficient matrix is A' .

$$\text{So } X = (A')^{-1}B = (A^{-1})'B = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 14 + 16 - 30 \\ 7 + 8 - 20 \\ 21 + 16 - 40 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$x = 0, y = -5, z = -3$ (check: $-2(-5) - 3 = 7, 0 + 5 + 3 = 8, 0 + 10 = 10$ ✓)

Q33(b) Singular $\Rightarrow$ determinant = 0. Expand along C_3 :

$$\sin 3x \begin{vmatrix} -7 & 4 \\ -11 & 7 \end{vmatrix} - \cos 2x \begin{vmatrix} 3 & -1 \\ -11 & 7 \end{vmatrix} + 2 \begin{vmatrix} 3 & -1 \\ -7 & 4 \end{vmatrix} = 0$$

$$\sin 3x(-49 + 44) - \cos 2x(21 - 11) + 2(12 - 7) = 0 \Rightarrow -5 \sin 3x - 10 \cos 2x + 10 = 0$$

$$\sin 3x + 2 \cos 2x - 2 = 0 \Rightarrow (3 \sin x - 4 \sin^3 x) + 2(1 - 2 \sin^2 x) - 2 = 0$$

$$3 \sin x - 4 \sin^3 x - 4 \sin^2 x = 0 \Rightarrow \sin x(4 \sin^2 x + 4 \sin x - 3) = 0 \Rightarrow \sin x(2 \sin x - 1)(2 \sin x + 3) = 0$$

$$\sin x = 0 \text{ or } \sin x = \frac{1}{2} \text{ (} \sin x = -\frac{3}{2} \text{ is impossible).}$$

$$\text{In } \left[0, \frac{\pi}{2}\right]: x = 0 \text{ or } x = \frac{\pi}{6}$$

Q34 $\frac{dx}{dt} = -\sin t, \frac{dy}{dt} = -m \sin mt$, so $\frac{dy}{dx} = \frac{m \sin mt}{\sin t}$.

$$\sin t = \sqrt{1 - x^2} \text{ and } \sin mt = \sqrt{1 - y^2}, \text{ so } \sqrt{1 - x^2} \frac{dy}{dx} = m \sqrt{1 - y^2}.$$

$$\text{Square: } (1 - x^2) \left(\frac{dy}{dx}\right)^2 = m^2(1 - y^2)$$

Differentiate w.r.t. x : $(1 - x^2) \cdot 2 \frac{dy}{dx} \frac{d^2y}{dx^2} - 2x \left(\frac{dy}{dx} \right)^2 = -2m^2y \frac{dy}{dx}$

Divide by $2 \frac{dy}{dx}$: $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = -m^2y$, i.e. $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$. Hence proved.

Q35

Direction ratios: (2, 3, 4) and (5, 2, 1). Since $\frac{2}{5} \neq \frac{3}{2}$, the lines are **not parallel**.

General points: $L_1: (2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$; $L_2: (5\mu + 4, 2\mu + 1, \mu)$.

Equate: $2\lambda + 1 = 5\mu + 4 \dots (1)$; $3\lambda + 2 = 2\mu + 1 \dots (2)$; $4\lambda + 3 = \mu \dots (3)$

From (3) in (1): $2\lambda + 1 = 20\lambda + 15 + 4 \Rightarrow -18\lambda = 18 \Rightarrow \lambda = -1$, so $\mu = -1$.

Check (2): $3(-1) + 2 = -1$ and $2(-1) + 1 = -1 \checkmark$. The lines intersect.

Point of intersection: $\lambda = -1$ in L_1 : **(-1, -1, -1)**

Q36

(i) Ten subscribers leave for every ₹ 1, so **10x** subscribers discontinue.

(ii) Subscribers = $5000 - 10x$, fee = $300 + x$; $R(x) = (300 + x)(5000 - 10x)$

(iii) (a) $R'(x) = (5000 - 10x) - 10(300 + x) = 2000 - 20x$; $R'(x) = 0 \Rightarrow x = 100$.

$R''(x) = -20 < 0$, so R is maximum at **x = 100** (fee ₹ 400).

OR (iii) (b) $R'(x) = 2000 - 20x > 0$ for $x < 100$ and < 0 for $x > 100$.

R(x) is **increasing in (0, 100)** and **decreasing in (100, 5000)**.

Q37

(i) **₹ 3,00,000, ₹ 2,00,000 or ₹ 50,000** (or nothing).

(ii) (a) Winning at least ₹ 2,00,000 means the first prize or a second prize: $1 + 2 = 3$ tickets out of 1,00,000.

$$P = \frac{3}{100000}$$

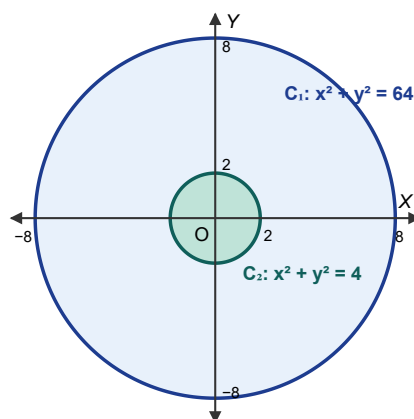
$$\text{OR (ii) (b) Winning tickets} = 1 + 2 + 3 = 6. P(\text{no prize}) = 1 - \frac{6}{100000} = \frac{99994}{100000} = \frac{49997}{50000}$$

(iii) Let E_1 : wins on the first ticket, $P(E_1) = \frac{6}{100000}$; E_2 : wins on the second, $P(E_2) = \frac{1}{100000}$ (independent).

$$P(\text{exactly one}) = P(E_1)P(E_2') + P(E_1')P(E_2) = \frac{6}{10^5} \cdot \frac{99999}{10^5} + \frac{99994}{10^5} \cdot \frac{1}{10^5} = \frac{699988}{10^{10}}$$

Q38

(i) Two concentric circles with centre at the origin, radii 8 and 2:



(ii) $C_1: y = \pm\sqrt{64 - x^2}$; $C_2: y = \pm\sqrt{4 - x^2}$.

(iii) (a) By symmetry, area $= 4 \int_0^8 \sqrt{64 - x^2} dx = 4 \left[\frac{x}{2} \sqrt{64 - x^2} + \frac{64}{2} \sin^{-1} \frac{x}{8} \right]_0^8$
 $= 4 \left[0 + 32 \cdot \frac{\pi}{2} - 0 \right] = 64\pi$ **sq units**

OR (iii) (b) Area $= 4 \int_0^2 \sqrt{4 - x^2} dx = 4 \left[\frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} \right]_0^2 = 4 \left[2 \cdot \frac{\pi}{2} \right] = 4\pi$ **sq units**



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