CLASS 12 · CBSE MATHEMATICS (041)

Mathematics Solved Board Papers

Chapter-wise and year-wise CBSE board questions with complete step-by-step solutions

10 years · 2016–2026

- ✓ Part A: every question sorted chapter-wise, grouped by marks, with full solutions
- ✓ Part B: 17 complete board papers, each followed by its solutions
- ✓ Board weightage table and most-asked topics for every chapter
- ✓ Every step of every proof, integral and calculation shown the way the board awards marks
- ✓ Questions on topics out of the board syllabus are flagged

WHAT YOU GET

10 years of CBSE Maths papers. Every question solved. Sorted chapter-wise.

138	2439	10	13	38
board papers	solved questions	years · 2016–26	chapters	diagrams

✗ Questions scattered across years	→ ✓ All in one place, chapter-wise
✗ Don't know what to study first	→ ✓ Weightage + most-asked topics
✗ One-line answers online	→ ✓ Full solutions, board-style
✗ Old books carry deleted topics	→ ✓ Topics out of the syllabus clearly marked

PART A — Chapter-wise

- ✓ Weightage table
- ✓ Most-asked topics
- ✓ Every question + solution

PART B — Full papers

- ✓ Exactly as set by CBSE
- ✓ Solutions after each paper
- ✓ Timed 3-hour practice

2026 · 65/1/1	2026 · 65/1/2	2026 · 65/1/3	2026 · 65/2/1	2025 · 65/1/1
2025 · 65/2/1	2024 · 65/1/1	2024 · 65/2/1	2023 · 65/1/1	2023 · 65/2/1
2022 · 65/1/1	2022 · 65/2/1	2020 · 65/1/1	2020 · 65/2/1	2019 · 65/1/1
2019 · 65/2/1	2018 · 65/1	2017 · 65/1	2016 · 65/1/N	

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CLASS 6–12 · ALL SUBJECTS

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Made for board toppers



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How to use this book

Part A puts every board question of a chapter together, from 1-mark MCQs to 5-mark long answers. Finish a chapter in your notes, then solve its Part A questions without looking at the solutions. Check your answer, mark the ones you got wrong, and repeat them a week later.

Each question shows where it was asked, for example **CBSE 2026 · 65/1/1 · Q13**. Use the weightage table and the most-asked topics chart at the start of each chapter to decide what to revise first.

Part B gives full papers. In the last month before the exam, sit with a clock and write one paper in 3 hours, then check it with the solutions that follow the paper.

Solutions are written the way the board awards marks: every step written, the property or formula used named at that step, and the final answer stated clearly. Write your answers the same way.

Contents

Part A · Chapter-wise questions with solutions

1. Relations and Functions 96 Qs
2. Inverse Trigonometric Functions 132 Qs
3. Matrices 163 Qs
4. Determinants 171 Qs
5. Continuity and Differentiability 269 Qs
6. Application of Derivatives 194 Qs
7. Integrals 362 Qs
8. Application of Integrals 110 Qs
9. Differential Equations 200 Qs
10. Vector Algebra 211 Qs
11. Three Dimensional Geometry 192 Qs
12. Linear Programming 121 Qs
13. Probability 218 Qs

Part B · Full board papers, each followed by its solutions

1. CBSE 2026 · 65/1/1 solutions
2. CBSE 2026 · 65/1/2 solutions
3. CBSE 2026 · 65/1/3 solutions
4. CBSE 2026 · 65/2/1 solutions
5. CBSE 2025 · 65/1/1 solutions
6. CBSE 2025 · 65/2/1 solutions
7. CBSE 2024 · 65/1/1 solutions
8. CBSE 2024 · 65/2/1 solutions
9. CBSE 2023 · 65/1/1 solutions
10. CBSE 2023 · 65/2/1 solutions
11. CBSE 2022 · 65/1/1 solutions
12. CBSE 2022 · 65/2/1 solutions
13. CBSE 2020 · 65/1/1 solutions
14. CBSE 2020 · 65/2/1 solutions
15. CBSE 2019 · 65/1/1 solutions
16. CBSE 2019 · 65/2/1 solutions
17. CBSE 2018 · 65/1 solutions
18. CBSE 2017 · 65/1 solutions
19. CBSE 2016 · 65/1/N solutions

2439 questions in all. Questions are taken from the CBSE Class 12 Maths board papers and the solutions are our own, checked against the official marking schemes where they are available. Questions on topics no longer in the current syllabus (binary operations, composition and inverse of functions, elementary operations, proofs by properties of determinants, Rolle's and mean value theorems, tangents and normals, approximations, definite integral as a limit of a sum, formation of differential equations, scalar triple product, the plane and others) are left out of Part A for papers before 2022 and marked in Part B. Where a topic is named only in CBSE's practicals or objectives, or was asked in a recent paper, its questions stay in Part A so that nothing you may need is missing.

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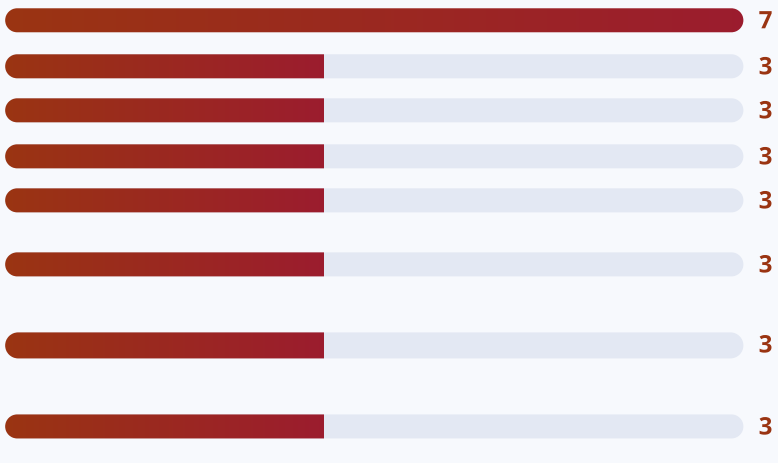
CHAPTER 1

Relations and Functions

Board weightage 2016–2026

Year	2016	2017	2018	2019	2020	2022	2023	2024	2025	2026
Questions per paper	0.2	0.7	2	1.6	2.1	–	1.7	2.6	2.7	2.4
Marks per paper	1	4	12	6	6	–	6	8	7	7

Most asked topics



96 different questions from 105 papers. Repeated questions are shown once with every year they were asked.

1 mark: MCQ and Assertion–Reason

CBSE 2026 · 65/2/1 · Q1 — 1 mark

A relation R on set A = {1, 2, 3} defined as R = {(1, 1), (2, 2), (1, 2)} is

(A) Reflexive only
(B) Reflexive and Transitive
(C) Symmetric and Transitive
(D) Transitive only

Solution

Answer: (D) Transitive only

Not reflexive: $(3,3) \notin R$. Not symmetric: $(1,2) \in R$ but $(2,1) \notin R$. Transitive: the only chains are $(1,1), (1,2) \Rightarrow (1,2) \in R$ and $(1,2), (2,2) \Rightarrow (1,2) \in R$.

CBSE 2026 · 65/2/2 · Q12 — 1 mark

A relation R on set A = {1, 2, 3} defined as R = {(1, 2), (2, 1), (2, 2)} is

(A) Reflexive only
(B) Reflexive and Transitive
(C) Symmetric and Transitive
(D) Symmetric only

Solution

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Answer: (D) Symmetric only

Not reflexive: (1, 1) and (3, 3) are missing. Symmetric: (1, 2) ∈ R and (2, 1) ∈ R; (2, 2) is its own reverse. Not transitive: (1, 2) ∈ R and (2, 1) ∈ R, but (1, 1) ∉ R.

CBSE 2026 · 65/2/3 · Q5 — 1 mark

A relation R on set A = {1, 2, 3} is defined as R = {(1, 3), (3, 3), (1, 1), (2, 2), (3, 1)} is

(A) only reflexive and symmetric
(B) reflexive only
(C) only reflexive and transitive
(D) reflexive, symmetric and transitive

Solution

Answer: (D)

Reflexive: (1, 1), (2, 2), (3, 3) ∈ R. Symmetric: (1, 3) and (3, 1) are both in R. Transitive: (1, 3), (3, 1) ⇒ (1, 1) ∈ R; (3, 1), (1, 3) ⇒ (3, 3) ∈ R; every other pair chain involves a pair (a, a) and is trivially satisfied. So R is an equivalence relation.

CBSE 2026 · 65/4/1 · Q20 | CBSE 2026 · 65/4/2 · Q20 | CBSE 2026 · 65/4/3 · Q20 — asked 3 times — 1 mark

Assertion (A) : A function f : N → N given by $f(x) = x^3 + 2$, ∀ x ∈ N is one-one but not onto.
Reason (R) : Since ∀ y ∈ N (Codomain), there does not exist $x = (y-2)^{1/3}$ in N (Domain) such that $f(x) = x^3 + 2 = y$.

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

Solution

Answer: (A) (as per the CBSE marking scheme)

A is true: $x_1^3 + 2 = x_2^3 + 2 \Rightarrow x_1 = x_2$ (one-one), and values such as y = 1, 2, 4 have no pre-image in N (not onto). R gives the reason for "not onto": for y in N, $x = (y-2)^{1/3}$ is generally not a natural number. (Strictly, it fails for some y, not every y — e.g. y = 3 has x = 1 — but the board accepted R as the explanation.)

CBSE 2026 · 65/5/1 · Q19 | CBSE 2026 · 65/5/2 · Q19 | CBSE 2026 · 65/5/3 · Q19 — asked 3 times — 1 mark

Assertion (A) : A relation R on the set {1, 2, 3} defined as R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} is an equivalence relation.
Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

Solution

Answer: (A)

Reflexive: (1,1), (2,2), (3,3) ∈ R. Symmetric: (1,2) and (2,1) both present. Transitive: (1,2),(2,1) ⇒ (1,1) ∈ R; (2,1),(1,2) ⇒ (2,2) ∈ R; other chains are trivial. So A is true, and R (the definition) explains it.

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CBSE 2025 · 65/1/1 · Q19 | CBSE 2025 · 65/1/2 · Q19 | CBSE 2025 · 65/1/3 · Q20 **asked 3 times** **1 mark**

Assertion (A) : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

Solution

Answer: (D)

f is one-one, but not onto Z: e.g. $3x - 5 = 0$ gives $x = \frac{5}{3} \notin Z$, so 0 has no pre-image. A is false; R is the definition of a bijection (true).

CBSE 2025 · 65/4/1 · Q3 | CBSE 2025 · 65/4/2 · Q5 | CBSE 2025 · 65/4/3 · Q8 **asked 3 times** **1 mark**

For real x, let $f(x) = x^3 + 5x + 1$. Then :

(A) f is one-one but not onto on R

(B) f is onto on R but not one-one

(C) f is one-one and onto on R

(D) f is neither one-one nor onto on R

Solution

Answer: (C)

$f'(x) = 3x^2 + 5 > 0$, so f is strictly increasing (one-one). Being a cubic, it is continuous and $f(x) \to \pm\infty$ as $x \to \pm\infty$, so it takes every real value (onto).

CBSE 2025 · 65/4/1 · Q9 | CBSE 2025 · 65/4/2 · Q10 | CBSE 2025 · 65/4/3 · Q5 **asked 3 times** **1 mark**

If f : N → W is defined as $f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases}$, then f is :

(A) injective only

(B) surjective only

(C) a bijection

(D) neither surjective nor injective

Solution

Answer: (B) surjective only

Not injective: $f(1) = f(3) = 0$. Surjective: 0 = f(1), and every $w \geq 1$ in W equals $f(2w)$.

CBSE 2025 · 65/5/1 · Q20 | CBSE 2025 · 65/5/2 · Q20 | CBSE 2025 · 65/5/3 · Q20 **asked 3 times** **1 mark**

Assertion (A) : Let A = {x ∈ R : –1 ≤ x ≤ 1}. If f : A → A be defined as f(x) = x^2, then f is not an onto function.

Reason (R) : If y = –1 ∈ A, then $x = \pm\sqrt{-1} \notin A$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is *not* the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Solution

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Answer: (A)

$x^2 \geq 0$, so the range of f is [0, 1] ≠ A. In particular –1 ∈ A has no pre-image, because $x^2 = -1$ gives $x = \pm\sqrt{-1}$, which is not a real number in A. Both are true and R explains A.

CBSE 2025 · 65/6/1 · Q20 | CBSE 2025 · 65/6/2 · Q20 | CBSE 2025 · 65/6/3 · Q20 **asked 3 times** 1 mark

Assertion (A) : Let $f(x) = e^x$ and g(x) = log x. Then $(f + g)x = e^x + \log x$ where domain of (f + g) is R.

Reason (R) : Dom(f + g) = Dom(f) ∩ Dom(g).

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is *not* the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Solution

Answer: (D) Assertion (A) is false, but Reason (R) is true.

R is the rule for the domain of a sum. Dom(f) = R, Dom(g) = (0, ∞), so Dom(f + g) = (0, ∞) ≠ R. Hence A is false and R is true.

CBSE 2025 · 65/7/2 · Q10 1 mark

If R be a relation defined as aRb iff |a – b| > 0 a, b ∈ $\mathbb{R}$ then R is :

(A) reflexive

(B) symmetric

(C) transitive

(D) symmetric and transitive

Solution

Answer: (B) symmetric

Not reflexive: |a – a| = 0, not > 0. Symmetric: |a – b| = |b – a|, so aRb ⇒ bRa. Not transitive: 1R2 and 2R1, but 1 R 1 is false.

CBSE 2024 · 65/1/1 · Q1 1 mark

A function $f : R_+ \to R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Solution

Answer: (A)

$4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$: one-one. For $x \geq 0$, $f(x) \geq 3$, so values below 3 (e.g. 0) have no pre-image: not onto R.

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CBSE 2024 · 65/1/2 · Q13 — 1 mark

A relation R defined on set A = {x : x ∈ Z and 0 ≤ x ≤ 10} as R = {(x, y) : x = y} is given to be an equivalence relation. The number of equivalence classes is :

(A) 1
(B) 2
(C) 10
(D) 11

Solution

Answer: (D) 11

A = {0, 1, 2, ..., 10} has 11 elements. Under the identity relation each element is related only to itself, so each class is a single element {a}: 11 equivalence classes.

CBSE 2024 · 65/1/3 · Q7 — 1 mark

A relation R defined on a set of human beings as R = {(x, y) : x is 5 cm shorter than y} is :

(A) reflexive only
(B) reflexive and transitive
(C) symmetric and transitive
(D) neither transitive, nor symmetric, nor reflexive

Solution

Answer: (D)

Not reflexive: no one is 5 cm shorter than himself. Not symmetric: if x is 5 cm shorter than y, then y is 5 cm taller than x, not shorter. Not transitive: if x is 5 cm shorter than y and y is 5 cm shorter than z, then x is 10 cm (not 5 cm) shorter than z.

CBSE 2024 · 65/2/1 · Q2 | CBSE 2024 · 65/2/3 · Q8 — asked 2 times — 1 mark

Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

(A) one-one
(B) onto
(C) bijective
(D) neither one-one nor onto

Solution

Answer: (C) bijective

$f(x) = (3x + 1)^2 - 6$. For $x \geq 0$, $3x + 1 \geq 1$ and f is strictly increasing (one-one). Its values run from $f(0) = -5$ upward without bound, covering $[-5, \infty)$ (onto): for $y \geq -5$, $x = \dfrac{\sqrt{y + 6} - 1}{3} \geq 0$.

CBSE 2024 · 65/2/2 · Q14 — 1 mark

Which of the following statements is ***not*** true about equivalence classes A_i (i = 1, 2, ... n) formed by an equivalence relation R defined on a set A ?

(A) $\bigcup_{i=1}^{n} A_i = A$

(B) $A_i \cap A_j \neq \phi$, i ≠ j

(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$

(D) All elements of A_i are related to each other, for all i

Solution

Answer: (B)

Equivalence classes partition A: they cover A (A true), distinct classes are disjoint, i.e. $A_i \cap A_j = \phi$ for i ≠ j (so B is false), a common element forces the classes to be equal (C true), and all elements of a class are related to each other (D true).

CBSE 2024 · 65/3/1 · Q2 — 1 mark

Let R_+ denote the set of all non-negative real numbers. Then the function $f : R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is :

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Solution

Answer: (A)

For $x_1, x_2 \geq 0$: $x_1^2 + 1 = x_2^2 + 1 \Rightarrow x_1 = x_2$, so f is one-one. But $f(x) \geq 1$, so values in [0, 1) (e.g. 0) have no pre-image: not onto.

CBSE 2024 · 65/3/2 · Q8 — 1 mark

A function f : R → A defined as $f(x) = x^2 + 1$ is onto, if A is :

(A) (– ∞, ∞)

(B) (1, ∞)

(C) [1, ∞)

(D) [–1, ∞)

Solution

Answer: (C) [1, ∞)

f is onto when the codomain equals the range. $x^2 \geq 0$ takes every non-negative value, so $x^2 + 1$ takes every value in [1, ∞) (1 at x = 0).

CBSE 2024 · 65/3/3 · Q8 — 1 mark

Let Z denote the set of integers, then function f : Z → Z defined as $f(x) = x^3 - 1$ is :

(A) both one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

Solution

Answer: (B)

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$x_1^3 - 1 = x_2^3 - 1 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$: one-one.

For y = 1 we need $x^3 = 2$, which has no integer solution, so 1 has no pre-image in Z: not onto.

CBSE 2024 · 65/4/1 · Q19 | CBSE 2024 · 65/4/2 · Q20 | CBSE 2024 · 65/4/3 · Q19 **asked 3 times** **1 mark**

Assertion (A) : The relation R = {(x, y) : (x + y) is a prime number and x, y ∈ N} is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Solution

Answer: (C)

(2, 2): 2 + 2 = 4 is not prime, so (2, 2) ∉ R and R is not reflexive (A true). But 2n = 2 is prime for n = 1, so "2n is composite for all n" is false.

CBSE 2024 · 65/5/1 · Q1 | CBSE 2024 · 65/5/2 · Q6 | CBSE 2024 · 65/5/3 · Q9 **asked 3 times** **1 mark**

A function f : $\mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

(A) injective but not surjective.

(B) surjective but not injective.

(C) both injective and surjective.

(D) neither injective nor surjective.

Solution

Answer: (D)

f(1) = 2 = f(3), so f is not injective. $f(x) = (x-2)^2 + 1 \geq 1$, so values below 1 (e.g. 0) have no pre-image: not surjective.

CBSE 2023 · 65/1/2 · Q20 **1 mark**

Assertion (A) : The number of onto functions from a set P containing 5 elements to a set Q containing 2 elements is 30.

Reason (R) : Number of onto functions from a set containing m elements to a set containing n elements is n^m.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is *not* the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Solution

Answer: (c)

There are $2^5 = 32$ functions from P to Q; exactly 2 of them are not onto (all elements sent to the same element). So the number of onto functions is 32 – 2 = 30 (A true). n^m counts *all* functions, not onto ones (R false).

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CBSE 2023 · 65/5/1 · Q1 | CBSE 2023 · 65/5/2 · Q2 **asked 2 times** **1 mark**

Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4

(c) 0 (d) 8

Solution

Answer: (b) 4

A reflexive relation must contain (3, 3) and (5, 5). Each of the other two pairs (3, 5), (5, 3) may or may not be included: $2^2 = 4$ relations.

CBSE 2023 · 65/5/3 · Q1 **1 mark**

Let R be a relation in the set N given by R = {(a, b) : a = b – 2, b > 6}. Then

(a) (8, 7) ∈ R (b) (6, 8) ∈ R

(c) (3, 8) ∈ R (d) (2, 4) ∈ R

Solution

Answer: (b)

(6, 8): b = 8 > 6 and a = 8 – 2 = 6 ✓. (8, 7): 8 ≠ 5. (3, 8): 3 ≠ 6. (2, 4): b = 4 is not > 6.

CBSE 2020 · 65/1/1 · Q11 | CBSE 2020 · 65/1/2 · Q14 | CBSE 2020 · 65/1/3 · Q14 **asked 3 times** **1 mark**

A relation R in a set A is called _____, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

Solution

Symmetric

CBSE 2020 · 65/2/1 · Q1 | CBSE 2020 · 65/2/2 · Q7 | CBSE 2020 · 65/2/3 · Q7 **asked 3 times** **1 mark**

The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is

(A) symmetric and transitive, but not reflexive (B) reflexive and symmetric, but not transitive

(C) symmetric, but neither reflexive nor transitive (D) an equivalence relation

Solution

Answer: (C)

(2, 2), (3, 3) ∉ R: not reflexive. (1, 2) and (2, 1) both in R: symmetric. (2, 1), (1, 2) ∈ R but (2, 2) ∉ R: not transitive.

CBSE 2020 · 65/2/1 · Q5 **1 mark**

Let A = {1, 3, 5}. Then the number of equivalence relations in A containing (1, 3) is

(A) 1 (B) 2

(C) 3 (D) 4

Solution

Answer: (B) 2

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1 and 3 must be in the same class: either {1, 3}, {5} or {1, 3, 5} (the universal relation).

CBSE 2020 · 65/2/2 · Q5 — 1 mark

The function f : R → [–1, 1] defined by f(x) = cos x is

(A) both one-one and onto
(B) not one-one, but onto
(C) one-one, but not onto
(D) neither one-one, nor onto

Solution

Answer: (B)

cos 0 = cos 2π = 1, so f is not one-one. Every value in [–1, 1] is taken by cos x, so f is onto [–1, 1].

CBSE 2020 · 65/5/1 · Q11 | CBSE 2020 · 65/5/3 · Q15 — asked 2 times — 1 mark

A relation in a set A is called _____ relation, if each element of A is related to itself.

Solution

Reflexive

CBSE 2020 · 65/5/2 · Q11 — 1 mark

A relation R on a set A is called _____, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for $a_1, a_2, a_3 \in A$.

Solution

Transitive

2 marks: very short answer

CBSE 2026 · 65/2/1 · Q21(a) | CBSE 2026 · 65/2/2 · Q24(a) | CBSE 2026 · 65/2/3 · Q22(a) — asked 3 times — 2 marks

Check whether $f : R - \{3\} \to R$ defined as $f(x) = \dfrac{x-2}{x-3}$ is onto or not.

Solution

Let $y = \dfrac{x-2}{x-3}$: $xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2 \Rightarrow x = \dfrac{3y-2}{y-1}$.

This is not defined for $y = 1$, i.e. no x gives $f(x) = 1$ (indeed $\dfrac{x-2}{x-3} = 1$ gives $-2 = -3$, impossible).

Range $= R - \{1\} \neq R$ (codomain), so **f is not onto**.

CBSE 2026 · 65/2/1 · Q21(b) | CBSE 2026 · 65/2/2 · Q24(b) | CBSE 2026 · 65/2/3 · Q22(b) — asked 3 times — 2 marks

Check whether $f : Z \times Z \to Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.

Solution

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Let $f(x_1, y_1) = f(x_2, y_2)$: $(2y_1, 3x_1) = (2y_2, 3x_2) \Rightarrow 2y_1 = 2y_2$ and $3x_1 = 3x_2 \Rightarrow y_1 = y_2$, $x_1 = x_2$.

So $(x_1, y_1) = (x_2, y_2)$: **f is injective**.

CBSE 2026 · 65/3/1 · Q23(b) | CBSE 2026 · 65/3/2 · Q21(b) | CBSE 2026 · 65/3/3 · Q24(b) **asked 3 times** **2 marks**

A relation R on A = {1, 2, 3} is defined as R = {(1, 1) (3, 3), (1, 2)}. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set {1, 2, 3}.

Solution

R is **not symmetric**: (1, 2) ∈ R but (2, 1) ∉ R.

For an equivalence relation we need reflexivity, so (2, 2) must be added, and symmetry, so (2, 1) must be added. Then $R \cup R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$, which is also transitive.

$$R_1 = \{(2,2), (2,1)\}$$

CBSE 2025 · 65/7/1 · Q21 | CBSE 2025 · 65/7/2 · Q23 | CBSE 2025 · 65/7/3 · Q25 **asked 3 times** **2 marks**

Let f : A → B be defined by $f(x) = \dfrac{x-2}{x-3}$, where A = R – {3} and B = R – {1}. Discuss the bijectivity of the function.

Solution

One-one: $\dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3} \Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6 \Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$.

Onto: for y ∈ B, $y = \dfrac{x-2}{x-3} \Rightarrow xy - 3y = x - 2 \Rightarrow x = \dfrac{3y-2}{y-1}$, which is real since y ≠ 1, and x ≠ 3 (as 3y – 2 ≠ 3y – 3). So x ∈ A and f(x) = y.

Hence f is one-one and onto, i.e. a bijection.

CBSE 2023 · 65/1/1 · Q21(a) | CBSE 2023 · 65/1/2 · Q24(a) | CBSE 2023 · 65/1/3 · Q22(a) **asked 3 times** **2 marks**

A function f : A → B defined as f(x) = 2x is both one-one and onto. If A = {1, 2, 3, 4}, then find the set B.

Solution

f is onto, so B equals the range: $f(1) = 2$, $f(2) = 4$, $f(3) = 6$, $f(4) = 8$.

B = {2, 4, 6, 8}

CBSE 2020 · 65/4/1 · Q21(a) | CBSE 2020 · 65/4/2 · Q21(a) | CBSE 2020 · 65/4/3 · Q24(a) **asked 3 times** **2 marks**

Check if the relation R on the set A = {1, 2, 3, 4, 5, 6} defined as R = {(x, y) : y is divisible by x} is (i) symmetric (ii) transitive

Solution

(i) **Not symmetric**: (1, 2) ∈ R (2 is divisible by 1) but (2, 1) ∉ R.

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(ii) **Transitive**: if y = kx and z = my (k, m ∈ N), then z = (mk)x, so (x, y), (y, z) ∈ R ⇒ (x, z) ∈ R.

CBSE 2020 · 65/5/1 · Q21(b) | CBSE 2020 · 65/5/2 · Q26(b) | CBSE 2020 · 65/5/3 · Q22(b) **asked 3 times** **2 marks**

Check if the relation R in the set $\mathbb{R}$ of real numbers defined as R = {(a, b) : a < b} is (i) symmetric, (ii) transitive

Solution

(i) **Not symmetric**: (1, 2) ∈ R since 1 < 2, but (2, 1) ∉ R.

(ii) **Transitive**: if a < b and b < c, then a < c, so (a, b), (b, c) ∈ R ⇒ (a, c) ∈ R.

3 marks: short answer

CBSE 2026 · 65/4/1 · Q26(a) | CBSE 2026 · 65/4/2 · Q28(a) | CBSE 2026 · 65/4/3 · Q30(a) **asked 3 times** **3 marks**

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function f : A → B is defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.

Solution

One-one: $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3) \Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$. So f is one-one.

Onto: let $y \in B$ $(y \neq 1)$. $y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2 \Rightarrow x = \frac{3y-2}{y-1}$, defined since $y \neq 1$. Also $x \neq 3$ (else $3y - 2 = 3y - 3$, impossible), so $x \in A$, and $f(x) = y$. So f is onto.

Hence f is one-one and onto (bijective).

CBSE 2026 · 65/4/1 · Q26(b) | CBSE 2026 · 65/4/2 · Q28(b) | CBSE 2026 · 65/4/3 · Q30(b) **asked 3 times** **3 marks**

Let n be a fixed positive integer. A relation R is defined in set Z such that R = {(x, y) : (x – y) is divisible by n, x, y ∈ Z}. Determine if R is an equivalence relation.

Solution

Reflexive: $x - x = 0 = 0 \cdot n$, so $(x, x) \in R$ for all $x \in Z$.

Symmetric: if $x - y = kn$, then $y - x = (-k)n$, so $(x, y) \in R \Rightarrow (y, x) \in R$.

Transitive: if $x - y = kn$ and $y - z = mn$, then $x - z = (k + m)n$, so $(x, z) \in R$.

Hence R is an equivalence relation.

CBSE 2025 · 65/2/1 · Q27 **3 marks**

Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n ∈ N." Find whether R is reflexive, symmetric and transitive or not.

Solution

Reflexive: every natural number is a multiple of itself ($m = 1 \cdot m$), so mRm. R is reflexive.

Not symmetric: 8R2 (8 is a multiple of 2) but 2 is not a multiple of 8.

Transitive: if mRn and nRp then $m = kn$ and $n = lp$ for natural k, l, so $m = (kl)p$: mRp.

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R is reflexive and transitive but not symmetric.

CBSE 2025 · 65/2/2 · Q27 — 3 marks

Prove that f : N → N defined as f(x) = ax + b (a, b ∈ N) is one-one but not onto.

Solution

One-one: $f(x_1) = f(x_2) \Rightarrow ax_1 + b = ax_2 + b \Rightarrow a(x_1 - x_2) = 0 \Rightarrow x_1 = x_2$ (as $a \neq 0$).

Not onto: for $x \in N$, $f(x) = ax + b \geq a + b \geq 2$, since $a, b \geq 1$. So the natural number 1 (in the codomain) has no pre-image. (Equivalently, $y = ax + b$ gives $x = \frac{y-b}{a}$, which need not be a natural number.) Hence f is one-one but not onto.

CBSE 2025 · 65/2/3 · Q27 — 3 marks

Let R be a relation on set of real numbers $\mathbb{R}$ defined as {(x, y) : x – y + $\sqrt{3}$ is an irrational number, x, y ∈ $\mathbb{R}$}. Verify R for reflexivity, symmetry and transitivity.

Solution

Reflexive: $x - x + \sqrt{3} = \sqrt{3}$ is irrational, so $(x, x) \in R$ for every x. R is reflexive.

Not symmetric: $(\sqrt{3}, 0)$: $\sqrt{3} - 0 + \sqrt{3} = 2\sqrt{3}$ is irrational, so it is in R; but $(0, \sqrt{3})$: $0 - \sqrt{3} + \sqrt{3} = 0$ is rational, so it is not in R.

Not transitive: $(\sqrt{3}, 1)$: $2\sqrt{3} - 1$ irrational ∈ R; $(1, 2\sqrt{3})$: $1 - 2\sqrt{3} + \sqrt{3} = 1 - \sqrt{3}$ irrational ∈ R; but $(\sqrt{3}, 2\sqrt{3})$: $\sqrt{3} - 2\sqrt{3} + \sqrt{3} = 0$ rational ∉ R.

So R is reflexive but neither symmetric nor transitive.

CBSE 2025 · 65/4/1 · Q26(a) | CBSE 2025 · 65/4/2 · Q27(a) | CBSE 2025 · 65/4/3 · Q28(a) — asked 3 times — 3 marks

If f : R^+ → R is defined as $f(x) = \log_a x$ (a > 0 and a ≠ 1), prove that f is a bijection. (R^+ is a set of all positive real numbers.)

Solution

One-one: $\log_a x_1 = \log_a x_2 \Rightarrow a^{\log_a x_1} = a^{\log_a x_2} \Rightarrow x_1 = x_2$.

Onto: for any $y \in R$, take $x = a^y > 0$, so $x \in R^+$ and $f(x) = \log_a a^y = y$.

Hence f is a bijection.

CBSE 2025 · 65/4/1 · Q26(b) | CBSE 2025 · 65/4/2 · Q27(b) | CBSE 2025 · 65/4/3 · Q28(b) — asked 3 times — 3 marks

Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as R = {(x, y) : x + y = 6, x ∈ A, y ∈ B}. (i) Write all elements of R. (ii) Is R a function ? Justify. (iii) Determine domain and range of R.

Solution

(i) R = {(1, 5), (2, 4)}. (3 would need y = 3 ∉ B.)

(ii) R is **not a function** from A to B: the element 3 ∈ A has no image.

(iii) Domain = {1, 2}; range = {4, 5}.

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CBSE 2025 · 65/6/1 · Q28(a) | CBSE 2025 · 65/6/2 · Q30(a) | CBSE 2025 · 65/6/3 · Q29(a) asked 3 times 3 marks

A student wants to pair up natural numbers in such a way that they satisfy the equation 2x + y = 41, x, y ∈ N. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

Solution

y = 41 – 2x ≥ 1 ⇒ x ≤ 20. R = {(1, 39), (2, 37), (3, 35), ..., (19, 3), (20, 1)}.

Domain = {1, 2, 3, ..., 20}; Range = {1, 3, 5, ..., 39}.

Not reflexive: (1, 1) ∉ R, since 2 + 1 ≠ 41.

Not symmetric: (1, 39) ∈ R but (39, 1) ∉ R (78 + 1 ≠ 41).

Not transitive: (11, 19) ∈ R (22 + 19 = 41) and (19, 3) ∈ R (38 + 3 = 41), but (11, 3) ∉ R (22 + 3 ≠ 41).

Hence R is not an equivalence relation.

CBSE 2025 · 65/6/1 · Q28(b) | CBSE 2025 · 65/6/2 · Q30(b) | CBSE 2025 · 65/6/3 · Q29(b) asked 3 times 3 marks

Show that the function f : N → N, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$ is a bijection.

Solution

One-one: let $f(n_1) = f(n_2)$.

Both even: $n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$. Both odd: $n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$.

n_1 odd, n_2 even: $n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$, impossible since an even number minus an odd number is odd. So f is one-one.

Onto: take any m ∈ N. If m is odd, m + 1 is even and $f(m+1) = m$. If m is even, m – 1 is odd (and ≥ 1) and $f(m-1) = m$. Every m has a pre-image, so f is onto.

Hence f is a bijection.

CBSE 2025 · 65/7/1 · Q26(a) | CBSE 2025 · 65/7/2 · Q27(a) | CBSE 2025 · 65/7/3 · Q30(a) asked 3 times 3 marks

Show that the function f : R → R defined by $f(x) = 4x^3 - 5$, ∀ x ∈ R is one-one and onto.

Solution

One-one: $f(x_1) = f(x_2) \Rightarrow 4x_1^3 - 5 = 4x_2^3 - 5 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$ (the cube function is one-one on R).

Onto: for any y ∈ R, take $x = \left(\frac{y+5}{4}\right)^{1/3} \in R$. Then $f(x) = 4 \cdot \frac{y+5}{4} - 5 = y$.

Hence f is one-one and onto.

CBSE 2025 · 65/7/1 · Q26(b) | CBSE 2025 · 65/7/2 · Q27(b) | CBSE 2025 · 65/7/3 · Q30(b) asked 3 times 3 marks

Let R be a relation defined on a set N of natural numbers such that R = {(x, y) : xy is a square of a natural number, x, y ∈ N}. Determine if the relation R is an equivalence relation.

Solution

Reflexive: $x \cdot x = x^2$ is a perfect square, so (x, x) ∈ R.

Symmetric: if xy is a perfect square, so is yx = xy. So (x, y) ∈ R ⇒ (y, x) ∈ R.

Transitive: let $xy = a^2$ and $yz = b^2$ (a, b ∈ N). Then $xy \cdot yz = a^2b^2 \Rightarrow xz = \left(\frac{ab}{y}\right)^2$. So xz is a natural number whose square root $\frac{ab}{y}$ is rational; a natural number with a rational square root is a perfect square. Hence (x, z) ∈ R.

R is reflexive, symmetric and transitive, so it is an equivalence relation.

CBSE 2024 · 65/1/1 · Q26(a) | CBSE 2024 · 65/1/2 · Q30(a) | CBSE 2024 · 65/1/3 · Q28(a) **asked 3 times** **3 marks**

A relation R on set A = {1, 2, 3, 4, 5} is defined as R = {(x, y) : $|x^2 - y^2| < 8$}. Check whether the relation R is reflexive, symmetric and transitive.

Solution

Reflexive: $|x^2 - x^2| = 0 < 8$ for every x, so $(x, x) \in R$.

Symmetric: $|y^2 - x^2| = |x^2 - y^2|$, so $(x, y) \in R \Rightarrow (y, x) \in R$.

Not transitive: $(1, 2) \in R$ ($|1 - 4| = 3$) and $(2, 3) \in R$ ($|4 - 9| = 5$), but $(1, 3) \notin R$ ($|1 - 9| = 8$, not < 8).

R is reflexive and symmetric but not transitive.

CBSE 2024 · 65/1/1 · Q26(b) | CBSE 2024 · 65/1/2 · Q30(b) | CBSE 2024 · 65/1/3 · Q28(b) **asked 3 times** **3 marks**

A function f is defined from R → R as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

Solution

$a + b = 1, 2a + b = 3 \Rightarrow a = 2, b = -1$: $f(x) = 2x - 1$.

One-one: $2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$.

Onto: for any $y \in R$, $x = \frac{y + 1}{2} \in R$ and $f(x) = y$.

f is one-one and onto.

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4 marks: case-based / source-based

CBSE 2026 · 65/5/1 · Q36 | CBSE 2026 · 65/5/2 · Q38 | CBSE 2026 · 65/5/3 · Q38 **asked 3 times** **4 marks**

A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that : Girl students belong to Set A = {G_1, G_2, G_3, G_4}, Boy students belong to Set B = {B_1, B_2, B_3}.

(i) How many relations are possible from Set A → Set B ? (1)

(ii) Let R be a relation from A → B such that R = {(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)}. Is R an injective function ? Justify your answer. (1)

(iii) (a) Let the relation R from A → A be such that R = {(x, y), x, y ∈ A, x and y are students from the same colony in the city}. Verify if R is an equivalence relation. (2)

OR

(iii) (b) Verify if any function f : B → A is bijective. Give reason to support your answer. (2)

Solution

(i) $n(A \times B) = 4 \times 3 = 12$, so the number of relations $= 2^{12} = 4096$.

(ii) No. R is not even a function: G_1 has two images (B_1 and B_2). (Also G_2 and G_3 share the image B_2.)

(iii) (a) Reflexive: every x is from the same colony as itself. Symmetric: if x and y are from the same colony, so are y and x. Transitive: if x, y are from the same colony and y, z are from the same colony, then x, z are from the same colony. Hence R is an equivalence relation.

(iii) (b) No function f : B → A can be bijective: B has 3 elements and A has 4, so at most 3 elements of A can be images; f cannot be onto.

CBSE 2025 · 65/1/1 · Q37 | CBSE 2025 · 65/1/2 · Q38 | CBSE 2025 · 65/1/3 · Q36 **asked 3 times** **4 marks**

A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} : R_1 = {(2, 3), (3, 2)}; R_2 = {(1, 2), (1, 3), (3, 2)}; R_3 = {(1, 2), (2, 1), (1, 1)}; R_4 = {(1, 1), (1, 2), (3, 3), (2, 2)}; R_5 = {(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)}.

(i) Identify the relation which is reflexive, transitive but not symmetric. (1)

(ii) Identify the relation which is reflexive and symmetric but not transitive. (1)

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. (2)

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ? (2)

Solution

(i) R_4: contains (1, 1), (2, 2), (3, 3) (reflexive); (1, 2) without (2, 1) (not symmetric); every chain gives (1, 2), which is present (transitive).

(ii) R_5: reflexive and symmetric, but (1, 2), (2, 3) ∈ R_5 while (1, 3) ∉ R_5, so not transitive.

(iii) (a) R_1 **and** R_3. R_1: symmetric; not reflexive; (2, 3), (3, 2) but (2, 2) ∉ R_1 — not transitive. R_3: symmetric; (2, 2) ∉ R_3 — not reflexive; (2, 1), (1, 2) but (2, 2) ∉ R_3 — not transitive.

OR (iii) (b) Reflexivity needs (1, 1), (2, 2), (3, 3); symmetry needs (2, 1), (3, 1), (2, 3). With these all nine pairs are present (A × A), which is transitive.

Pairs to add: **(1, 1), (2, 2), (3, 3), (2, 1), (3, 1), (2, 3)**

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CBSE 2025 · 65/2/1 · Q36 | CBSE 2025 · 65/2/2 · Q38 | CBSE 2025 · 65/2/3 · Q36 **asked 3 times** **4 marks**

A school is organizing a debate competition with participants as speakers S = {S_1, S_2, S_3, S_4} and these are judged by judges J = {J_1, J_2, J_3}. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as R = {(x, y) : speaker x is judged by judge y, x ∈ S, y ∈ J}.

(i) How many relations can be there from S to J ? (1)

(ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)}. Check if it is bijective. (1)

(iii) (a) How many one-one functions can be there from set S to set J ? (2)

OR

(iii) (b) Another student considers a relation R_1 = {(S_1, S_2), (S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. (2)

Solution

(i) $S \times J$ has $4 \times 3 = 12$ elements; relations = subsets: $2^{12} = 4096$

(ii) $f(S_2) = f(S_3) = J_2$, so f is not one-one: **not bijective** (it is onto).

(iii) (a) A one-one function from a 4-element set into a 3-element set is impossible (two speakers must share a judge): **0**

OR (iii) (b) Add **(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)**. R_1 then is reflexive, and still not symmetric since (S_1, S_2) ∈ R_1 but (S_2, S_1) ∉ R_1.

CBSE 2025 · 65/5/1 · Q37 | CBSE 2025 · 65/5/2 · Q38 | CBSE 2025 · 65/5/3 · Q36 **asked 3 times** **4 marks**

Case Study – 2

Let A be the set of 30 students of class XII in a school. Let f : A → N, N is a set of natural numbers such that function f(x) = Roll Number of student x.

On the basis of the given information, answer the following :

(i) Is f a bijective function ? [1]

(ii) Give reasons to support your answer to (i). [1]

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}. List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. [2]

OR

(iii) (b) Let R be a relation defined by R = {(x, y) : x, y are Roll Numbers of students such that $y = x^3$}. List the elements of R. Is R a function ? Justify your answer. [2]

Solution

(Roll numbers are 1, 2, ..., 30.)

(i) No, f is not bijective.

(ii) Different students have different roll numbers, so f is one-one; but the range {1, 2, ..., 30} ≠ codomain N (e.g. 31 has no pre-image), so f is not onto, hence not bijective.

(iii)(a) R = {(1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21), (8, 24), (9, 27), (10, 30)}.

Not reflexive: (1, 1) ∉ R. Not symmetric: (1, 3) ∈ R but (3, 1) ∉ R. Not transitive: (1, 3), (3, 9) ∈ R but (1, 9) ∉ R.

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(iii)(b) R = {(1, 1), (2, 8), (3, 27)}. R is not a function (from the set of roll numbers), because the elements 4, 5, ..., 30 have no image.

CBSE 2024 · 65/5/1 · Q36 | CBSE 2024 · 65/5/2 · Q37 | CBSE 2024 · 65/5/3 · Q38 **asked 3 times** **4 marks**

(a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by $R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation y = 3x + 2, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

Solution

(a)(i) If $l_1 \parallel l_2$ then $l_2 \parallel l_1$: R is symmetric.

(ii) If $l_1 \parallel l_2$ and $l_2 \parallel l_3$, then $l_1 \parallel l_3$: R is transitive.

(iii) Lines parallel to y = 3x + 2 have the same slope 3: the set is {y = 3x + c : c ∈ R}.

(b) If $l_1 \perp l_2$ then $l_2 \perp l_1$: S is symmetric. But if $l_1 \perp l_2$ and $l_2 \perp l_3$, then $l_1 \parallel l_3$ (in a plane), not perpendicular: S is not transitive.

CBSE 2023 · 65/5/1 · Q36 | CBSE 2023 · 65/5/2 · Q37 | CBSE 2023 · 65/5/3 · Q36 **asked 3 times** **4 marks**

Case Study – I

An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = {b_1, b_2, b_3} and G = {g_1, g_2}, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let R : B → B be defined by R = {(x, y) : x and y are students of the same sex}. Check if R is an equivalence relation.

OR

(III) A function f : B → G be defined by f = {(b_1, g_1), (b_2, g_2), (b_3, g_1)}. Check if f is bijective. Justify your answer.

Solution

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(I) B × G has 3 × 2 = 6 elements, so there are $2^6 = 64$ relations from B to G.

(II) Each of the 3 boys can be sent to either of the 2 girls: $2^3 = 8$ functions.

(III) All elements of B are boys, so R = B × B. It is reflexive, symmetric and transitive: an equivalence relation.

(III) OR: f(b_1) = f(b_3) = g_1 with $b_1 \neq b_3$, so f is not one-one (it is onto, since both g_1 and g_2 are images). Hence f is not bijective.

CBSE 2020 · 65/1/1 · Q27 | CBSE 2020 · 65/1/2 · Q32 | CBSE 2020 · 65/1/3 · Q32 **asked 3 times** **4 marks**

Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d ∈ N. Show that R is an equivalence relation.

Solution

Reflexive: $ab = ba$, so (a, b) R (a, b).

Symmetric: $ad = bc \Rightarrow cb = da$, so (c, d) R (a, b).

Transitive: (a, b) R (c, d) and (c, d) R (e, f) give $ad = bc$ and $cf = de$. Multiply: $adcf = bcde \Rightarrow af = be$ (cancelling $cd \neq 0$), so (a, b) R (e, f).

Hence R is an equivalence relation.

CBSE 2020 · 65/2/1 · Q27(a) | CBSE 2020 · 65/2/2 · Q30(a) | CBSE 2020 · 65/2/3 · Q30(a) **asked 3 times** **4 marks**

Check whether the relation R in the set N of natural numbers given by R = {(a, b) : a is divisor of b} is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

Solution

Reflexive: every a divides itself.

Not symmetric: 2 divides 4 but 4 does not divide 2.

Transitive: a | b and b | c ⇒ b = ka, c = lb = (kl)a ⇒ a | c.

Since it is not symmetric, **R is not an equivalence relation**.

CBSE 2020 · 65/3/1 · Q27(a) | CBSE 2020 · 65/3/2 · Q31(a) | CBSE 2020 · 65/3/3 · Q31(a) **asked 3 times** **4 marks**

Show that the function f : (– ∞, 0) → (– 1, 0) defined by $f(x) = \dfrac{x}{1+|x|}$, x ∈ (– ∞, 0) is one-one and onto.

Solution

For x < 0, |x| = –x, so $f(x) = \dfrac{x}{1-x}$.

One-one: $\dfrac{x_1}{1-x_1} = \dfrac{x_2}{1-x_2} \Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$.

Onto: for y ∈ (–1, 0), solve $y = \dfrac{x}{1-x}$: $x = \dfrac{y}{1+y}$. Since 1 + y ∈ (0, 1) and y < 0, x < 0, so x ∈ (–∞, 0), and $f(x) = \dfrac{y/(1+y)}{1/(1+y)} = y$.

Hence f is one-one and onto.

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CBSE 2020 · 65/3/1 · Q27(b) | CBSE 2020 · 65/3/2 · Q31(b) | CBSE 2020 · 65/3/3 · Q31(b) **asked 3 times** **4 marks**

Show that the relation R in the set A = {1, 2, 3, 4, 5, 6} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation.

Solution

Reflexive: |a – a| = 0 is divisible by 2.

Symmetric: |a – b| = |b – a|, so (a, b) ∈ R ⇒ (b, a) ∈ R.

Transitive: if a – b = 2m and b – c = 2n, then a – c = 2(m + n), so (a, c) ∈ R.

Hence R is an equivalence relation. (Its classes are {1, 3, 5} and {2, 4, 6}.)

CBSE 2020 · 65/4/1 · Q27 | CBSE 2020 · 65/4/3 · Q28 **asked 2 times** **4 marks**

Prove that the relation R on Z, defined by R {(x, y) : (x – y) is divisible by 5} is an equivalence relation.

Solution

Reflexive: x – x = 0 = 5 × 0, so (x, x) ∈ R for every x ∈ Z.

Symmetric: if x – y = 5m (m ∈ Z), then y – x = 5(–m), so (y, x) ∈ R.

Transitive: if x – y = 5m and y – z = 5n, then x – z = 5(m + n), so (x, z) ∈ R.

Hence R is an equivalence relation.

CBSE 2020 · 65/4/2 · Q31 **4 marks**

Show that the function f : R → R defined by $f(x) = \dfrac{x}{x^2+1}$, ∀ x ∈ R is neither one-one nor onto.

Solution

Not one-one: $f(2) = \frac{2}{5}$ and $f\left(\frac{1}{2}\right) = \frac{1/2}{5/4} = \frac{2}{5}$, but $2 \neq \frac{1}{2}$.

Not onto: suppose f(x) = 1 for some real x: $x = x^2 + 1 \Rightarrow x^2 - x + 1 = 0$, whose discriminant is 1 – 4 < 0, so there is no real x. Thus 1 ∈ R has no pre-image.

(Indeed the range is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.) Hence f is neither one-one nor onto.

CBSE 2019 · 65/1/1 · Q13(a) | CBSE 2019 · 65/1/2 · Q20(a) | CBSE 2019 · 65/1/3 · Q15(a) **asked 3 times** **4 marks**

Show that the relation R on R defined as R = {(a, b) : a ≤ b}, is reflexive, and transitive but not symmetric.

Solution

Reflexive: $a \leq a$ for every real a, so (a, a) ∈ R.

Transitive: $a \leq b$ and $b \leq c \Rightarrow a \leq c$.

Not symmetric: (1, 2) ∈ R since $1 \leq 2$, but (2, 1) ∉ R.

CBSE 2019 · 65/1/1 · Q13(b) | CBSE 2019 · 65/1/2 · Q20(b) | CBSE 2019 · 65/1/3 · Q15(b) **asked 3 times** **4 marks**

Prove that the function f : N → N, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of f : N → S, where S is range of f.

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Solution

One-one: $x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1 \Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$; since $x_1 + x_2 + 1 > 0$, $x_1 = x_2$.

Not onto: $f(x) \geq 3$ for $x \in N$, so 1 and 2 have no pre-image.

Inverse: $y = x^2 + x + 1 \Rightarrow x = \dfrac{-1 + \sqrt{4y - 3}}{2}$, so $f^{-1}(y) = \dfrac{\sqrt{4y - 3} - 1}{2}$, $y \in S$.

CBSE 2019 · 65/2/1 · Q13(a) | CBSE 2019 · 65/2/2 · Q15(a) | CBSE 2019 · 65/2/3 · Q16(a) asked 3 times 4 marks

Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

Solution

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}.

Not reflexive: (1, 1) ∉ R. **Not symmetric**: (1, 2) ∈ R, (2, 1) ∉ R. **Not transitive**: (1, 2), (2, 3) ∈ R but (1, 3) ∉ R.

CBSE 2019 · 65/3/1 · Q13(a) | CBSE 2019 · 65/3/2 · Q13(a) | CBSE 2019 · 65/3/3 · Q16(a) asked 3 times 4 marks

Show that the relation R on the set Z of all integers, given by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

Solution

Reflexive: a – a = 0 is divisible by 2.

Symmetric: if a – b = 2m, then b – a = 2(–m), so (b, a) ∈ R.

Transitive: if a – b = 2m and b – c = 2n, then a – c = 2(m + n), so (a, c) ∈ R.

Hence R is an equivalence relation.

CBSE 2019 · 65/4/1 · Q15(a) | CBSE 2019 · 65/4/2 · Q19(a) | CBSE 2019 · 65/4/3 · Q18(a) asked 3 times 4 marks

Let A = R – {2} and B = R – {1}. If f : A → B is a function defined by $f(x) = \dfrac{x - 1}{x - 2}$, show that f is one-one and onto. Hence, find f^{-1}.

Solution

One-one: $\dfrac{x_1 - 1}{x_1 - 2} = \dfrac{x_2 - 1}{x_2 - 2} \Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2 \Rightarrow x_1 = x_2$.

Onto: for y ∈ B, $y = \dfrac{x - 1}{x - 2} \Rightarrow xy - 2y = x - 1 \Rightarrow x = \dfrac{2y - 1}{y - 1}$, real since y ≠ 1, and x ≠ 2 (2y – 1 ≠ 2y – 2). So x ∈ A with f(x) = y.

Hence f is a bijection and $f^{-1} : B \to A$, $f^{-1}(x) = \dfrac{2x - 1}{x - 1}$.

CBSE 2019 · 65/4/1 · Q15(b) | CBSE 2019 · 65/4/2 · Q19(b) | CBSE 2019 · 65/4/3 · Q18(b) asked 3 times 4 marks

Show that the relation S in the set A = {x ∈ Z : 0 ≤ x ≤ 12} given by S = {(a, b) : a, b ∈ Z, |a – b| is divisible by 3} is an equivalence relation.

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Solution

Reflexive: |a – a| = 0 is divisible by 3.

Symmetric: |a – b| = |b – a|, so (a, b) ∈ S ⇒ (b, a) ∈ S.

Transitive: if a – b = 3m and b – c = 3n, then a – c = 3(m + n), so |a – c| is divisible by 3.

Hence S is an equivalence relation.

CBSE 2019 · 65/5/1 · Q17(a) | CBSE 2019 · 65/5/2 · Q19(a) | CBSE 2019 · 65/5/3 · Q14(a) asked 3 times 4 marks

Prove that the relation R in the set A = {1, 2, 3, 4, 5, 6, 7} given by R = {(a, b) : |a – b| is even} is an equivalence relation.

Solution

Reflexive: |a – a| = 0 is even, so (a, a) ∈ R.

Symmetric: |a – b| = |b – a|, so (a, b) ∈ R ⇒ (b, a) ∈ R.

Transitive: if a – b and b – c are even, then a – c = (a – b) + (b – c) is even, so (a, c) ∈ R.

Hence R is an equivalence relation. (Its classes are {1, 3, 5, 7} and {2, 4, 6}.)

CBSE 2019 · 65/5/1 · Q17(b) | CBSE 2019 · 65/5/2 · Q19(b) | CBSE 2019 · 65/5/3 · Q14(b) asked 3 times 4 marks

Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

Hence, find f^{-1}.

Solution

One-one: $\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4} \Rightarrow 24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 - 16x_2 + 18x_1 - 12 \Rightarrow 34x_1 = 34x_2 \Rightarrow x_1 = x_2$.

Onto (f : A → A): for y ≠ 2/3, $y = \frac{4x+3}{6x-4} \Rightarrow 6xy - 4y = 4x + 3 \Rightarrow x = \frac{4y+3}{6y-4}$, which is defined, and x ≠ 2/3 (since 3(4y + 3) = 2(6y – 4) would give 9 = –8). Also f(x) ≠ 2/3 for every x, for the same reason.

Hence f is a bijection and $f^{-1}(x) = \frac{4x+3}{6x-4}$ (f is its own inverse).

5 marks: long answer

CBSE 2026 · 65/1/1 · Q32(a) | CBSE 2026 · 65/1/2 · Q34(a) | CBSE 2026 · 65/1/3 · Q33(a) asked 3 times 5 marks

A relation R is defined on Z, the set of integers, as $R = \{(x, y) : |x - y|$ is divisible by a prime number 'p', $x, y \in Z\}$; check whether R is an equivalence relation or not.

Solution

Reflexive: for any $x \in Z$, $|x - x| = 0 = 0 \cdot p$ is divisible by p, so $(x, x) \in R$.

Symmetric: if $(x, y) \in R$, then $|x - y| = kp$ for some integer k. Since $|y - x| = |x - y| = kp$, $(y, x) \in R$.

Transitive: if $(x, y) \in R$ and $(y, z) \in R$, then $x - y = mp$ and $y - z = np$ for integers m, n (signs absorbed in m, n). Adding: $x - z = (m + n)p$, so $|x - z|$ is divisible by p and $(x, z) \in R$.

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R is reflexive, symmetric and transitive, so **R is an equivalence relation**.

CBSE 2026 · 65/1/1 · Q32(b) | CBSE 2026 · 65/1/2 · Q34(b) | CBSE 2026 · 65/1/3 · Q33(b) **asked 3 times** **5 marks**

A function $f : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto.

Solution

One-one: let $f(x_1) = f(x_2)$: $\frac{3x_1+2}{5x_1-3} = \frac{3x_2+2}{5x_2-3}$

$(3x_1+2)(5x_2-3) = (3x_2+2)(5x_1-3)$

$15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 - 9x_2 + 10x_1 - 6 \Rightarrow -19x_1 = -19x_2 \Rightarrow x_1 = x_2$. So f is one-one.

Onto: let $y \in R - \left\{\frac{3}{5}\right\}$ and $f(x) = y$: $3x + 2 = 5xy - 3y \Rightarrow x(5y-3) = 3y+2 \Rightarrow x = \frac{3y+2}{5y-3}$, which is a real number since $y \neq \frac{3}{5}$.

Also $x \neq \frac{3}{5}$: if $\frac{3y+2}{5y-3} = \frac{3}{5}$ then $15y + 10 = 15y - 9$, impossible. So $x \in R - \left\{\frac{3}{5}\right\}$ and $f(x) = y$; f is onto.

Hence f is one-one and onto.

CBSE 2026 · 65/3/1 · Q32 **5 marks**

Show that $f : R \to R$ defined as $f(x) = \frac{x}{\sqrt{1+x^2}}$ is one-one but not onto.

Solution

One-one: let $f(x_1) = f(x_2)$, i.e. $\frac{x_1}{\sqrt{1+x_1^2}} = \frac{x_2}{\sqrt{1+x_2^2}}$. Since the denominators are positive, x_1 and x_2 have the same sign. Squaring:

$$x_1^2(1+x_2^2) = x_2^2(1+x_1^2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2 \text{ (same sign)}$$

So f is one-one.

Not onto: let $y = \frac{x}{\sqrt{1+x^2}}$. Then $y^2 = \frac{x^2}{1+x^2} \Rightarrow x^2 = \frac{y^2}{1-y^2}$, which needs $y^2 < 1$. Also $|x| < \sqrt{1+x^2}$ always, so $|f(x)| < 1$. The range is $(-1, 1) \neq R$; e.g. y = 2 has no pre-image. Hence f is not onto.

CBSE 2026 · 65/3/2 · Q34 **5 marks**

Show that $f : R_+ \to [-5, \infty)$ given by $f(x) = 4x^2 + 4x - 5$ is both one-one and onto where $R_+ = [0, \infty)$. Also, find $p \in R_+$ such that $f(p) = 3$.

Solution

One-one: let $f(x_1) = f(x_2)$ for $x_1, x_2 \in R_+$:

$$4(x_1^2 - x_2^2) + 4(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)\,[4(x_1 + x_2) + 4] = 0$$

Since $x_1 + x_2 \geq 0$, the second factor is at least 4, so $x_1 = x_2$: f is one-one.

Onto: let $y \in [-5, \infty)$. $y = 4x^2 + 4x - 5 = (2x+1)^2 - 6 \Rightarrow 2x + 1 = \sqrt{y+6}$ (taking the positive root, as $2x + 1 \geq 1$) $\Rightarrow x = \dfrac{\sqrt{y+6} - 1}{2}$.

For $y \geq -5$, $\sqrt{y+6} \geq 1$, so $x \geq 0$, i.e. $x \in R_+$, and $f(x) = y$. Every element of the codomain has a pre-image: f is onto.

f(p) = 3: $4p^2 + 4p - 5 = 3 \Rightarrow p^2 + p - 2 = 0 \Rightarrow (p+2)(p-1) = 0 \Rightarrow p = 1$ (p = –2 $\notin R_+$).

CBSE 2026 · 65/3/3 · Q33 — 5 marks

Show that a function $f : R_+ \to A \subset N$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $R_+ = [0, \infty)$. Also, find if there exists $a \in R_+$ such that $f(a) = 7$. Justify.

Solution

One-one: let $f(x_1) = f(x_2)$, $x_1, x_2 \geq 0$:

$$4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)\,[4(x_1 + x_2) + 12] = 0$$

The bracket is at least 12, so $x_1 = x_2$: f is one-one.

Set A: $f(x) = 4x^2 + 12x + 15 = (2x+3)^2 + 6$. On $[0, \infty)$, $2x + 3 \geq 3$ and increases without bound, so f takes every value from $f(0) = 15$ upwards. For f to be onto, A must be the range:

$$A = [15, \infty) = \{y : y \geq 15\}$$

(Each $y \geq 15$ has the pre-image $x = \dfrac{\sqrt{y-6} - 3}{2} \geq 0$.) (The paper calls A a subset of N; for real $x \in R_+$ the range is the interval $[15, \infty)$. If the domain is taken as the natural numbers, A = {f(n) : n ∈ N} = {31, 55, 87, ...}.)

f(a) = 7: $4a^2 + 12a + 15 = 7 \Rightarrow a^2 + 3a + 2 = 0 \Rightarrow a = -1$ or $a = -2$; neither belongs to R_+. Also $7 < 15$, the least value of f. So **no such a exists**.

CBSE 2024 · 65/2/1 · Q32(a) | CBSE 2024 · 65/2/2 · Q35(a) asked 2 times — 5 marks

Show that a function $f : R \to R$ defined by $f(x) = \dfrac{2x}{1 + x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

Solution

Not one-one: $f(2) = \dfrac{4}{5}$ and $f\left(\dfrac{1}{2}\right) = \dfrac{1}{5/4} = \dfrac{4}{5}$, but $2 \neq \dfrac{1}{2}$.

Not onto: let $y = \dfrac{2x}{1 + x^2} \Rightarrow yx^2 - 2x + y = 0$. For real x (y ≠ 0), discriminant $4 - 4y^2 \geq 0 \Rightarrow -1 \leq y \leq 1$. So e.g. y = 2 has no pre-image; f is not onto R.

Range $= [-1, 1]$ (y = 0 from x = 0; y = ±1 from x = ±1). So f is onto for **A = [–1, 1]**.

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CBSE 2024 · 65/2/1 · Q32(b) | CBSE 2024 · 65/2/2 · Q35(b) **asked 2 times** **5 marks**

A relation R is defined on N × N (where N is the set of natural numbers) as : (a, b) R (c, d) ⇔ a – c = b – d. Show that R is an equivalence relation.

Solution

Note $a - c = b - d \Leftrightarrow a - b = c - d$.

Reflexive: $a - a = b - b = 0$, so (a, b) R (a, b).

Symmetric: if $a - c = b - d$ then $c - a = d - b$, so (c, d) R (a, b).

Transitive: (a, b) R (c, d) and (c, d) R (e, f) give $a - b = c - d$ and $c - d = e - f$, so $a - b = e - f$, i.e. $a - e = b - f$: (a, b) R (e, f).

Hence R is an equivalence relation.

CBSE 2024 · 65/2/3 · Q33(a) **5 marks**

Show that a function f : R → R defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which f(x) = 3.

Solution

Not one-one: f(0) = 1 and f(–1) = 1 – 1 + 1 = 1, but 0 ≠ –1.

Not onto: $f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$ for all x, so e.g. 0 ∈ R (codomain) has no pre-image.

f(x) = 3: $x^2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0 \Rightarrow x = 1$ or x = –2.

CBSE 2024 · 65/2/3 · Q33(b) **5 marks**

A relation R is defined on N × N (where N is the set of natural numbers) as $(a, b)\, R\, (c, d) \Leftrightarrow \frac{a}{c} = \frac{b}{d}$. Show that R is an equivalence relation.

Solution

For natural numbers, $\frac{a}{c} = \frac{b}{d} \iff ad = bc$.

Reflexive: ab = ba, so (a, b) R (a, b).

Symmetric: (a, b) R (c, d) ⇒ ad = bc ⇒ cb = da ⇒ (c, d) R (a, b).

Transitive: (a, b) R (c, d) and (c, d) R (e, f) ⇒ ad = bc and cf = de. Multiplying: $ad \cdot cf = bc \cdot de \Rightarrow af = be$ (cancelling cd ≠ 0), so (a, b) R (e, f).

Hence R is an equivalence relation.

CBSE 2024 · 65/3/1 · Q32 **5 marks**

A relation R on set A = {– 4, – 3, – 2, – 1, 0, 1, 2, 3, 4} be defined as R = {(x, y) : x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

Solution

Reflexive: x + x = 2x is divisible by 2, so (x, x) ∈ R for all x ∈ A.

Symmetric: if x + y is divisible by 2, so is y + x. So (x, y) ∈ R ⇒ (y, x) ∈ R.

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Transitive: let x + y = 2m and y + z = 2n. Then x + z = 2m + 2n – 2y = 2(m + n – y), divisible by 2. So (x, z) ∈ R.

Hence R is an equivalence relation.

[2] = {x ∈ A : x + 2 is even} = {x ∈ A : x is even} = {–4, –2, 0, 2, 4}.

CBSE 2024 · 65/3/2 · Q35 — 5 marks

A relation R on set A = {x : – 10 ≤ x ≤ 10, x ∈ Z} is defined as R = {(x, y) : (x – y) is divisible by 5}. Show that R is an equivalence relation. Also, write the equivalence class [5].

Solution

Reflexive: x – x = 0 is divisible by 5, so (x, x) ∈ R for every x ∈ A.

Symmetric: if x – y = 5m, then y – x = 5(–m), so (x, y) ∈ R ⇒ (y, x) ∈ R.

Transitive: if x – y = 5m and y – z = 5n, then x – z = 5(m + n), so (x, z) ∈ R.

Hence R is an equivalence relation.

[5] = {x ∈ A : x – 5 is divisible by 5} = {x ∈ A : x is a multiple of 5} = {–10, –5, 0, 5, 10}.

CBSE 2024 · 65/3/3 · Q33 — 5 marks

Let A = R – {3} and B = R – {a}. Find the value of 'a' such that the function f : A → B defined by $f(x) = \dfrac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not.

Solution

Range of f: $y = \dfrac{x-2}{x-3} \Rightarrow xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2 \Rightarrow x = \dfrac{3y-2}{y-1}$.

This gives a real x (and x ≠ 3, because $\frac{3y-2}{y-1} = 3$ would need –2 = –3) for every y ≠ 1, while y = 1 is impossible ($\frac{x-2}{x-3} = 1$ gives –2 = –3). So the range is R – {1}.

f is onto B exactly when B = range, i.e. **a = 1**.

One-one: $\dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3} \Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6 \Rightarrow x_1 = x_2$.

So f is one-one (hence a bijection from R – {3} to R – {1}).

CBSE 2024 · 65/4/1 · Q33(a) | CBSE 2024 · 65/4/2 · Q32(a) | CBSE 2024 · 65/4/3 · Q32(a) — asked 3 times — 5 marks

Let A = R – {5} and B = R – {1}. Consider the function f : A → B, defined by $f(x) = \dfrac{x-3}{x-5}$. Show that f is one-one and onto.

Solution

One-one: $\dfrac{x_1-3}{x_1-5} = \dfrac{x_2-3}{x_2-5} \Rightarrow x_1x_2 - 5x_1 - 3x_2 + 15 = x_1x_2 - 5x_2 - 3x_1 + 15 \Rightarrow -2x_1 = -2x_2 \Rightarrow x_1 = x_2$.

Onto: for y ∈ B (y ≠ 1), solve $y = \dfrac{x-3}{x-5}$: $xy - 5y = x - 3 \Rightarrow x = \dfrac{5y-3}{y-1}$. This is real (y ≠ 1) and x ≠ 5 (5y – 3 = 5y – 5 is impossible), so x ∈ A and f(x) = y.

Hence f is one-one and onto.

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CBSE 2024 · 65/4/1 · Q33(b) | CBSE 2024 · 65/4/2 · Q32(b) | CBSE 2024 · 65/4/3 · Q32(b) asked 3 times 5 marks

Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where a – b + $\sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.

Solution

Reflexive: a – a + √2 = √2 is irrational, so (a, a) ∈ S for all a. S is reflexive.

Not symmetric: (√2, 0): √2 – 0 + √2 = 2√2 is irrational, so (√2, 0) ∈ S; but (0, √2): 0 – √2 + √2 = 0 is rational, so (0, √2) ∉ S.

Not transitive: (√2, 1): 2√2 – 1 irrational ∈ S; (1, 2√2): 1 – 2√2 + √2 = 1 – √2 irrational ∈ S; but (√2, 2√2): √2 – 2√2 + √2 = 0 rational ∉ S.

So S is reflexive, but neither symmetric nor transitive.

CBSE 2023 · 65/1/1 · Q32 | CBSE 2023 · 65/1/2 · Q34 | CBSE 2023 · 65/1/3 · Q35 asked 3 times 5 marks

A relation R is defined on a set of real numbers R as R = {(x, y) : x·y is an irrational number}. Check whether R is reflexive, symmetric and transitive or not.

Solution

Not reflexive: $1 \cdot 1 = 1$ is rational, so $(1, 1) \notin R$.

Symmetric: $xy = yx$, so if xy is irrational, so is yx: $(x, y) \in R \Rightarrow (y, x) \in R$.

Not transitive: $(1, \sqrt{2}) \in R$ (product $\sqrt{2}$) and $(\sqrt{2}, 1) \in R$, but $(1, 1) \notin R$.

R is symmetric but neither reflexive nor transitive.

CBSE 2023 · 65/2/1 · Q35 | CBSE 2023 · 65/2/2 · Q34 | CBSE 2023 · 65/2/3 · Q32 asked 3 times 5 marks

A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

Solution

Onto: for any $y \in [0, 4]$, take $x = \sqrt{16 - y^2} \in [0, 4] \subset [-4, 4]$; then $f(x) = \sqrt{16 - (16 - y^2)} = y$.

Not one-one: $f(-4) = f(4) = 0$ (in general $f(-x) = f(x)$).

$f(a) = \sqrt{7} \Rightarrow 16 - a^2 = 7 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$

CBSE 2023 · 65/3/1 · Q32 5 marks

Show that a function f : $\mathbb{R} \to \mathbb{R}$ defined as $f(x) = \dfrac{5x - 3}{4}$ is both one-one and onto.

Solution

One-one: $\dfrac{5x_1 - 3}{4} = \dfrac{5x_2 - 3}{4} \Rightarrow 5x_1 = 5x_2 \Rightarrow x_1 = x_2$.

Onto: for any y ∈ $\mathbb{R}$, take $x = \dfrac{4y + 3}{5} \in \mathbb{R}$. Then $f(x) = \dfrac{(4y + 3) - 3}{4} = y$.

Hence f is one-one and onto.

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CBSE 2023 · 65/3/2 · Q34 — 5 marks

Prove that a function f : [0, ∞) → [– 5, ∞) defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

Solution

One-one: $f(x_1) = f(x_2) \Rightarrow 4(x_1^2 - x_2^2) + 4(x_1 - x_2) = 0 \Rightarrow 4(x_1 - x_2)(x_1 + x_2 + 1) = 0$.
For $x_1, x_2 \geq 0$, $x_1 + x_2 + 1 > 0$, so $x_1 = x_2$.

Onto: $f(x) = (2x + 1)^2 - 6$. For y ∈ [–5, ∞), solve $(2x + 1)^2 = y + 6$: $x = \dfrac{\sqrt{y + 6} - 1}{2}$.

Since y ≥ –5, $\sqrt{y + 6} \geq 1$, so x ≥ 0, i.e. x ∈ [0, ∞), and f(x) = y.

Hence f is one-one and onto.

CBSE 2023 · 65/3/3 · Q33 — 5 marks

Check whether a function f : $\mathbb{R} \to \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1 + x^2}$ is one-one and onto or not.

Solution

Not one-one: $f(2) = \frac{2}{5}$ and $f\left(\frac{1}{2}\right) = \frac{1/2}{5/4} = \frac{2}{5}$, but $2 \neq \frac{1}{2}$.

Onto: let $y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. For y = 0, x = 0 works. For y ≠ 0, $y = \frac{x}{1+x^2} \iff yx^2 - x + y = 0$, which has real roots iff $1 - 4y^2 \geq 0 \iff |y| \leq \frac{1}{2}$, true. So every y in the codomain has a pre-image.

Hence f is onto but not one-one.

CBSE 2023 · 65/4/1 · Q34(a) | CBSE 2023 · 65/4/2 · Q34(a) | CBSE 2023 · 65/4/3 · Q35(a) — asked 3 times — 5 marks

If N denotes the set of all natural numbers and R is the relation on N × N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation.

Solution

Dividing by abcd (≠ 0): $ad(b + c) = bc(a + d) \iff \dfrac{1}{c} + \dfrac{1}{b} = \dfrac{1}{d} + \dfrac{1}{a} \iff \dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c} - \dfrac{1}{d}$.

Reflexive: $\frac{1}{a} - \frac{1}{b} = \frac{1}{a} - \frac{1}{b}$, so (a, b) R (a, b).

Symmetric: if $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$, then $\frac{1}{c} - \frac{1}{d} = \frac{1}{a} - \frac{1}{b}$, so (c, d) R (a, b).

Transitive: if $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ and $\frac{1}{c} - \frac{1}{d} = \frac{1}{e} - \frac{1}{f}$, then $\frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f}$, i.e. (a, b) R (e, f).

Hence R is an equivalence relation.

CBSE 2023 · 65/4/1 · Q34(b) | CBSE 2023 · 65/4/2 · Q34(b) | CBSE 2023 · 65/4/3 · Q35(b) — asked 3 times — 5 marks

Let f : $\mathbb{R} - \left\{-\dfrac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \dfrac{4x}{3x + 4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

Solution

One-one: $\dfrac{4x_1}{3x_1 + 4} = \dfrac{4x_2}{3x_2 + 4} \Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2 \Rightarrow x_1 = x_2$.

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Onto? $y = \dfrac{4x}{3x+4} \Rightarrow 3xy + 4y = 4x \Rightarrow x(4-3y) = 4y \Rightarrow x = \dfrac{4y}{4-3y}$, which requires $y \neq \frac{4}{3}$.

For $y = \frac{4}{3}$ there is no x (it would need $0 = \frac{16}{3}$). So $\frac{4}{3} \in \mathbb{R}$ has no pre-image: f is **not onto** (its range is $\mathbb{R} - \{\frac{4}{3}\}$).

6 marks: long answer

CBSE 2018 · 65/1 · Q24(a) | CBSE 2018 · 65/2 · Q26(a) | CBSE 2018 · 65/3 · Q25(a) **asked 3 times** **6 marks**

Let A = {x ∈ Z : 0 ≤ x ≤ 12}. Show that R = {(a, b) : a, b ∈ A, |a – b| is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

Solution

Reflexive: $|a-a| = 0$ is divisible by 4. **Symmetric:** $|b-a| = |a-b|$. **Transitive:** $a - b = 4m$, $b - c = 4n \Rightarrow a - c = 4(m+n)$.

So R is an equivalence relation.

Elements related to 1: $|1-b|$ divisible by 4: **{1, 5, 9}**. Equivalence class **[2] = {2, 6, 10}**.

CBSE 2018 · 65/1 · Q24(b) | CBSE 2018 · 65/2 · Q26(b) | CBSE 2018 · 65/3 · Q25(b) **asked 3 times** **6 marks**

Show that the function f : R → R defined by $f(x) = \dfrac{x}{x^2+1}$, ∀ x ∈ R is neither one-one nor onto.

Also, if g : R → R is defined as g(x) = 2x – 1, find fog(x).

Solution

Not one-one: $f(2) = \frac{2}{5} = f\left(\frac{1}{2}\right)$.

Not onto: $y = \frac{x}{x^2+1} \Rightarrow yx^2 - x + y = 0$; real x needs $1 - 4y^2 \geq 0$, i.e. $|y| \leq \frac{1}{2}$. So e.g. y = 1 has no pre-image.

$$fog(x) = f(2x-1) = \frac{2x-1}{(2x-1)^2+1} = \frac{2x-1}{4x^2-4x+2}$$

CBSE 2017 · 65/1 · Q25(a) | CBSE 2017 · 65/2 · Q27(a) | CBSE 2017 · 65/3 · Q27(a) **asked 3 times** **6 marks**

Consider $f : R - \left\{-\dfrac{4}{3}\right\} \to R - \left\{\dfrac{4}{3}\right\}$ given by $f(x) = \dfrac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

Solution

One-one: $\dfrac{4x_1+3}{3x_1+4} = \dfrac{4x_2+3}{3x_2+4} \Rightarrow 16x_1 + 9x_2 = 9x_1 + 16x_2 \Rightarrow x_1 = x_2$.

Onto: $y = \dfrac{4x+3}{3x+4} \Rightarrow x = \dfrac{4y-3}{4-3y}$, defined for $y \neq \frac{4}{3}$ and never equal to $-\frac{4}{3}$. So f is bijective.

$f^{-1}(y) = \dfrac{4y-3}{4-3y}$; $f^{-1}(0) = -\dfrac{3}{4}$; $\dfrac{4x-3}{4-3x} = 2 \Rightarrow x = \dfrac{11}{10}$

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CBSE 2017 · 65/2/1 · Q25(a) | CBSE 2017 · 65/2/2 · Q26(a) | CBSE 2017 · 65/2/3 · Q26(a) **asked 3 times** **6 marks**

Let A = R – {3}, B = R – {1}. Let f : A → B be defined by $f(x) = \dfrac{x-2}{x-3}$, ∀ x ∈ A.

Show that f is bijective. Also, find

(i) x, if $f^{-1}(x) = 4$

(ii) $f^{-1}(7)$

Solution

One-one: $\dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3} \Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2 \Rightarrow x_1 = x_2$.

Onto: $y = \dfrac{x-2}{x-3} \Rightarrow x = \dfrac{3y-2}{y-1}$, which is defined for every $y \neq 1$ and is never 3. So f is bijective, with $f^{-1}(x) = \dfrac{3x-2}{x-1}$.

(i) $\dfrac{3x-2}{x-1} = 4 \Rightarrow 3x - 2 = 4x - 4 \Rightarrow$ **x = 2**

(ii) $f^{-1}(7) = \dfrac{21-2}{6} = \dfrac{19}{6}$

CBSE 2016 · 65/1/E · Q20 | CBSE 2016 · 65/2/E · Q26 | CBSE 2016 · 65/3/E · Q25 **asked 3 times** **6 marks**

Show that the relation R defind by (a, b) R (c, d) ⇒ a + d = b + c on the A × A, where A = {1, 2, 3, ..., 10} is an equivalence relation. Hence write the equivalence class [(3, 4)]; a, b, c, d ∈ A.

Solution

Reflexive: a + b = b + a, so (a, b) R (a, b).

Symmetric: (a, b) R (c, d) ⇒ a + d = b + c ⇒ c + b = d + a ⇒ (c, d) R (a, b).

Transitive: (a, b) R (c, d) and (c, d) R (e, f) give a + d = b + c and c + f = d + e. Adding: a + f = b + e, so (a, b) R (e, f).

Hence R is an equivalence relation.

[(3, 4)] = {(c, d) ∈ A × A : 3 + d = 4 + c} = {(c, c + 1)} = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10)}.

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CBSE BOARD PAPER

2026 · Q.P. Code 65/1/1

Time: 3 hours
Maximum marks: 80

Section A · Multiple choice and Assertion–Reason (1 mark each)

1. If $2\cos^{-1}x = y$, then 1

(A) $0 \le y \le \pi$ (B) $-\pi \le y \le \pi$

(C) $0 \le y \le 2\pi$ (D) $-\pi \le y \le 0$

2. Which of the following cannot be the order of a row-matrix ? 1

(A) 2×1 (B) 1×2

(C) 1×1 (D) $1 \times n$

3. Which of the following properties is/are true for two matrices of suitable orders ? (i) $(A+B)' = A' + B'$ (ii) $(A-B)' = B' - A'$ (iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar) 1

(A) (i) only (B) (i), (ii) and (iii)

(C) (i) and (ii) (D) (i) and (iv)

4. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then 1

(A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$

(C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$

5. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is 1

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

6. If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric ? 1

(A) AB (B) AB + BA

(C) $(A+B)^2$ (D) A – B

7. The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is 1

(A) –16 (B) –9

(C) 0 (D) 16

8. If $\int \frac{3ax}{b^2 + c^2x^2}\,dx = A\log|b^2 + c^2x^2| + K$, then the value of A is 1

(A) 3a (B) $\frac{3a}{2b^2}$

(C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$

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9. The value of $\int_{-1}^{1} \frac{x^3}{x^2+2|x|+1}\,dx$ is 1

(A) 0
(B) log 2
(C) 2 log 2
(D) $\frac{1}{2}\log 2$

10. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by 1

(A) 0
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{4}{3}$

11. The integrating factor of differential equation $R\frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is 1

(A) $e^{\int \frac{P}{Q}dy}$
(B) $e^{\int P\,dy}$
(C) $e^{\int \frac{P}{R}dy}$
(D) $e^{\int \frac{P}{R}dx}$

12. The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are 1

(A) 0, 1
(B) 1, 1
(C) 2, 1
(D) 1, not defined

13. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is 1

(A) 0
(B) 1
(C) $\frac{5}{2}$
(D) $-\frac{5}{2}$

14. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is 1

(A) 8
(B) –8
(C) 2
(D) $\frac{5}{2}$

15. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $|\vec{a} \cdot \vec{b}|$ 1

(A) $6\sqrt{3}$
(B) $8\sqrt{3}$
(C) $12\sqrt{3}$
(D) $3\sqrt{12}$

16. The length of perpendicular drawn from point (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is 1

(A) 2
(B) 5
(C) $\sqrt{74}$
(D) $\sqrt{78}$

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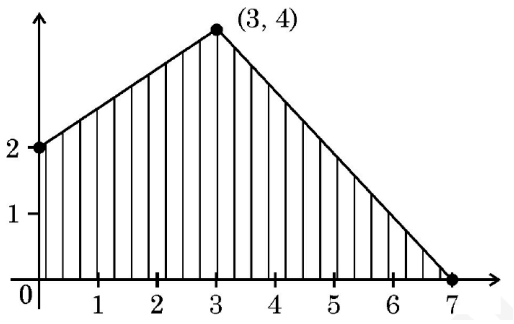
17. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below : 1



The maximum value of Z – minimum value of Z is

(A) 8 (B) 29

(C) 35 (D) 43

18. The degree of an objective function of a linear programming problem is 1

(A) 0 (B) 1

(C) 2 (D) Any natural number

19. Assertion (A) : In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$. 1

Reason (R) : For any two events A and B, $P(A|B) = \frac{P(A \cup B)}{P(B)}$

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

20. Assertion (A) : Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. 1

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

(A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.

Section B · Very short answer (2 marks each)

21. **(a)** Check whether function f(x) defined as $f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not ? 2

OR

(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. 2

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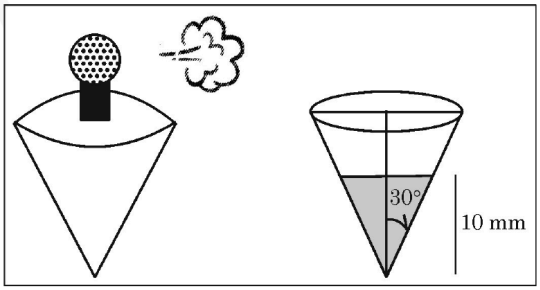
22. A room freshner bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of 1 mm^3/min. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$. 2



23. Find the vector of magnitude 14 in the direction of $\vec{QP}$, where P and Q are the points (1, 3, 2) and (–1, 0, 8) respectively. 2

24. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths. 2

25. **(a)** Simplify : $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right), 0 < x < \frac{\pi}{4}$. 2

OR

(b) Evaluate : $\tan\left(\sin^{-1} 1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$ 2

Section C · Short answer (3 marks each)

26. Evaluate : $\int_0^1 x \tan^{-1} x \, dx$. 3

27. **(a)** Find $\int \sqrt{\frac{x+2}{x-2}}\, dx$ 3

OR

(b) Find : $\int \frac{x^2}{(x^2+9)(x^2+16)}\, dx$ 3

28. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x|\, dx$, then show that $I_1 - 4I_2 = 0$. 3

29. **(a)** Find the general solution of the following differential equation : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ 3

OR

(b) Find the particular solution of the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$, given that $y(1) = -1$. 3

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30. Solve the following linear programming problem graphically : Minimize $Z = 13x - 15y$ subject to constraints $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$ 3

31. **(a)** Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball. 3

OR

(b) The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$. 3

Section D · Long answer (5 marks each)

32. **(a)** A relation R is defined on Z, the set of integers, as $R = \{(x, y) : |x - y|$ is divisible by a prime number 'p', $x, y \in Z\}$; check whether R is an equivalence relation or not. 5

OR

(b) A function $f : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x + 2}{5x - 3}$. Show that f is one-one and onto. 5

33. **(a)** If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations : $-2y + z = 7, 2x - y - z = 8, x - 2y = 10$ 5

OR

(b) If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$. 5

34. If $x = \cos t, y = \cos mt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$. 5

35. Check whether the lines given by $\frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4}$ and $\frac{x - 4}{5} = \frac{y - 1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting. 5

Section E · Case-based (4 marks each)

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36. *An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries. The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x.* 4

(i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? (1)

(ii) If R(x) denotes the total revenue collected after the increase of ₹ x in subscription fee, express R(x) as a function of x. (1)

(iii) (a) Find the value of x for which R(x) is maximum. (2)

OR

(iii) (b) Find the sub-intervals of (0, 5000) in which R(x) is increasing and decreasing. (2)

37. *In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each. A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000. Rohan bought one ticket.* 4

(i) What are the possible amounts, the person can win ? (1)

(ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? (2)

OR

(ii) (b) What is the probability that the person does not win any amount ? (2)

(iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot. (1)

38. *Roundabouts are often made on busy roads to ease the traffic and avoid red lights. One such round-about is made such that equation representing its boundary is given by* $C_1 : x^2 + y^2 = 64$*. There is a circular pond with a fountain in the middle of the roundabout whose equation is given by* $C_2 : x^2 + y^2 = 4$*.* 4

(i) Represent the given equations C_1 and C_2 with the help of a diagram. (1)

(ii) Express y as a function of x, (y = f(x)), for both C_1 and C_2. (1)

(iii) (a) Using integration find the area of region covered by the roundabout. (2)

OR

(iii) (b) Using integration, find the area of region covered by circular pond. (2)

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Q1 **Answer: (C)**

The principal value range of $\cos^{-1} x$ is $[0, \pi]$, so $0 \le \cos^{-1} x \le \pi$ gives $0 \le 2\cos^{-1} x \le 2\pi$, i.e. $0 \le y \le 2\pi$.

Q2 **Answer: (A)** 2×1

A row matrix has exactly one row, so its order is $1 \times n$. A 2×1 matrix has two rows (it is a column matrix).

Q3 **Answer: (D)**

$(A + B)' = A' + B'$ is true. $(A - B)' = A' - B'$, not $B' - A'$, so (ii) is false. $(AB)' = B'A'$ (order reversed), so (iii) is false. $(kAB)' = k(AB)' = kB'A'$, so (iv) is true.

Q4 **Answer: (B)**

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Δ_1 is a diagonal determinant: $\Delta_1 = 1 \times 2 \times 3 = 6$.

Expanding Δ_2 along R_1: $\Delta_2 = 0 - 2\begin{vmatrix} 1 & 0 \\ 0 & 6 \end{vmatrix} + 0 = -2(6) = -12$.

So $\Delta_2 = -12 = -2\Delta_1$.

Q5 **Answer: (B) $\frac{\pi}{4}$**

$\cos x \sin x - (-\cos x)\sin x = 2\sin x \cos x = \sin 2x = 1$, so $2x = \frac{\pi}{2}$, $x = \frac{\pi}{4}$.

Q6 **Answer: (D) A – B**

$A' = -A$, $B' = -B$. Then $(A - B)' = A' - B' = -A + B = -(A - B)$, so $A - B$ is skew symmetric.

(For (B): $(AB + BA)' = B'A' + A'B' = BA + AB$, which is symmetric.)

Q7 **Answer: (A) –16**

$f'(x) = 3x^2 - 12 = 0 \Rightarrow x = 2$ (in [0, 3]).

$f(0) = 0$, $f(2) = 8 - 24 = -16$, $f(3) = 27 - 36 = -9$. Least value $= -16$.

Q8 **Answer: (D)**

Put $t = b^2 + c^2x^2$, $dt = 2c^2x\,dx$: $\int \frac{3ax}{t} \cdot \frac{dt}{2c^2x} = \frac{3a}{2c^2}\log|t| + K$. So $A = \frac{3a}{2c^2}$.

Q9 **Answer: (A) 0**

$f(-x) = \frac{-x^3}{x^2 + 2|x| + 1} = -f(x)$, so the integrand is odd, and $\int_{-a}^{a} f(x)\,dx = 0$ for an odd function.

Q10 **Answer: (C) $\frac{2}{3}$**

$y = x^2$ for $x \geq 0$ and $y = -x^2$ for $x < 0$; the two parts are symmetric about the origin (one above, one below the x-axis). Area $= 2\int_0^1 x^2\,dx = 2 \times \frac{1}{3} = \frac{2}{3}$.

Q11 **Answer: (C)**

Divide by R: $\frac{dx}{dy} + \frac{P}{R}x = \frac{Q}{R}$, linear in x. IF $= e^{\int \frac{P}{R}dy}$.

Q12 **Answer: (B) 1, 1**

$\frac{d}{dx}(e^y) = e^y\frac{dy}{dx} = 0$. The highest derivative is $\frac{dy}{dx}$ (order 1), occurring to the first power (degree 1).

Q13 **Answer: (C) $\frac{5}{2}$**

Perpendicular: dot product $= 0$: $1(2) + 2(-p) + 3(1) = 0 \Rightarrow 5 - 2p = 0 \Rightarrow p = \frac{5}{2}$.

Q14 **Answer: (A) 8**

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$\vec{AB} = 3\hat{i} + (m+1)\hat{j} + 3\hat{k}$, $\vec{AC} = 4\hat{i} + 12\hat{j} + 4\hat{k}$. Collinear ⇒ components proportional: $\frac{3}{4} = \frac{m+1}{12} = \frac{3}{4} \Rightarrow m+1 = 9 \Rightarrow m = 8$.

Q15 **Answer: (C)** $12\sqrt{3}$

$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$: $144 + (\vec{a} \cdot \vec{b})^2 = 64 \times 9 = 576$, so $(\vec{a} \cdot \vec{b})^2 = 432$, $|\vec{a} \cdot \vec{b}| = \sqrt{432} = 12\sqrt{3}$.

Q16 **Answer: (C)** $\sqrt{74}$

The line passes through the origin with direction (1, 0, 0): it is the x-axis. The foot of the perpendicular from (2, 5, 7) is (2, 0, 0), so the length $= \sqrt{5^2 + 7^2} = \sqrt{74}$.

Q17 **Answer: (D) 43**

Corner points: (0, 0), (0, 2), (3, 4), (7, 0).

Z: (0, 0) → 0; (0, 2) → 14; (3, 4) → 15 + 28 = 43; (7, 0) → 35. Max = 43, min = 0, difference = 43.

Q18 **Answer: (B) 1**

The objective function $Z = ax + by$ is linear, so its degree is 1.

Q19 **Answer: (C)**

Odd outcomes {1, 3, 5}; primes among them {3, 5}. $P(\text{prime}|\text{odd}) = \frac{2/6}{3/6} = \frac{2}{3}$, so A is true.

R is false: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Q20 **Answer: (D)**

First line: $\frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r}$, direction (p, 1, r); second: (p′, 1, r′). Perpendicular ⇔ $pp' + 1 + rr' = 0$, i.e. $pp' + rr' = -1$, not 1. So A is false; R is true.

Q21(a) For $x < 3$, $|x-3| = -(x-3)$, so $f(x) = \frac{-(x-3)}{2(x-3)} = -\frac{1}{2}$.

LHL $= \lim_{x \to 3^-} f(x) = -\frac{1}{2}$.

RHL $= \lim_{x \to 3^+} \frac{x-6}{6} = \frac{3-6}{6} = -\frac{1}{2}$; $f(3) = \frac{3-6}{6} = -\frac{1}{2}$.

LHL = RHL = f(3), so **f is continuous at x = 3**.

Q21(b) Differentiate w.r.t. x: $\sqrt{3}\left(2x + 2y\frac{dy}{dx}\right) = 4y + 4x\frac{dy}{dx}$

$\frac{dy}{dx}(2\sqrt{3}y - 4x) = 4y - 2\sqrt{3}x \Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$

At $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$: numerator $= \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$; denominator $= \frac{3}{2} - 1 = \frac{1}{2}$.

$\frac{dy}{dx} = \sqrt{3}$

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Q22 Let the level be h and the radius of the surface r. Semi-vertical angle $\frac{\pi}{6}$: $r = h\tan\frac{\pi}{6} = \frac{h}{\sqrt{3}}$.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot \frac{h^2}{3} \cdot h = \frac{\pi h^3}{9}$

$\frac{dV}{dt} = \frac{\pi h^2}{3}\frac{dh}{dt} = -1$ (volume decreases)

At h = 10: $\frac{100\pi}{3}\frac{dh}{dt} = -1 \Rightarrow \frac{dh}{dt} = -\frac{3}{100\pi}$

The level is dropping at $\frac{3}{100\pi}$ **mm/min**.

Q23 $\vec{QP} = \vec{OP} - \vec{OQ} = (1+1)\hat{i} + (3-0)\hat{j} + (2-8)\hat{k} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$|\vec{QP}| = \sqrt{4+9+36} = 7$

Required vector $= 14 \times \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7} = 4\hat{i} + 6\hat{j} - 12\hat{k}$

Q24 Diagonals: $\vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{a} - \vec{b} = 2\hat{i} - 2\hat{j}$ (or $\vec{b} - \vec{a}$).

$|\vec{a} + \vec{b}| = \sqrt{16+4+16} = 6$; $|\vec{a} - \vec{b}| = \sqrt{4+4+0} = 2\sqrt{2}$

Q25(a) Divide numerator and denominator by $\cos 2x$:

$\tan^{-1}\left(\frac{1-\tan 2x}{1+\tan 2x}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - 2x\right)\right)$

For $0 < x < \frac{\pi}{4}$, $-\frac{\pi}{4} < \frac{\pi}{4} - 2x < \frac{\pi}{4}$, which lies in the principal range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

So the expression $= \frac{\pi}{4} - 2x$

Q25(b) $\sin^{-1} 1 = \frac{\pi}{2}$; $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\tan\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) = \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

Q26 By parts (first $\tan^{-1} x$, second x):

$I = \left[\frac{x^2}{2}\tan^{-1} x\right]_0^1 - \int_0^1 \frac{x^2}{2} \cdot \frac{1}{1+x^2}\,dx$

$= \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{2}\int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx = \frac{\pi}{8} - \frac{1}{2}\left[x - \tan^{-1} x\right]_0^1$

$= \frac{\pi}{8} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right) = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2}$

Q27(a) Multiply inside by $\frac{x+2}{x+2}$: $\sqrt{\frac{x+2}{x-2}} = \frac{x+2}{\sqrt{x^2-4}}$

$I = \int \frac{x}{\sqrt{x^2-4}}\,dx + 2\int \frac{dx}{\sqrt{x^2-4}}$

First: put $x^2 - 4 = t$, $2x\,dx = dt$: $\int \frac{dt}{2\sqrt{t}} = \sqrt{t} = \sqrt{x^2-4}$.

Second: standard form $\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x + \sqrt{x^2-a^2}|$.

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$$I = \sqrt{x^2-4} + 2\log\left|x + \sqrt{x^2-4}\right| + C$$

Q27(b) Put $x^2 = t$ (only for the partial fractions): $\dfrac{t}{(t+9)(t+16)} = \dfrac{A}{t+9} + \dfrac{B}{t+16}$

$t = A(t+16) + B(t+9)$. $t=-9$: $-9 = 7A \Rightarrow A = -\dfrac{9}{7}$; $t=-16$: $-16 = -7B \Rightarrow B = \dfrac{16}{7}$.

$$I = -\frac{9}{7}\int \frac{dx}{x^2+9} + \frac{16}{7}\int \frac{dx}{x^2+16} = -\frac{9}{7}\cdot\frac{1}{3}\tan^{-1}\frac{x}{3} + \frac{16}{7}\cdot\frac{1}{4}\tan^{-1}\frac{x}{4}$$

$$I = \frac{4}{7}\tan^{-1}\frac{x}{4} - \frac{3}{7}\tan^{-1}\frac{x}{3} + C$$

Q28 $1 + \cos 2x = 2\cos^2 x$: $I_1 = \dfrac{1}{2}\displaystyle\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = \frac{1}{2}\Big[\tan x\Big]_{-\pi/4}^{\pi/4} = \frac{1}{2}(1-(-1)) = 1$

$|x|$ is even: $I_2 = 2\displaystyle\int_0^{1/2} x\,dx = 2\left[\frac{x^2}{2}\right]_0^{1/2} = \frac{1}{4}$

$I_1 - 4I_2 = 1 - 4\times\dfrac{1}{4} = 0$. Hence proved.

Q29(a) $\dfrac{dy}{dx} = 1 + \dfrac{y}{x} + \left(\dfrac{y}{x}\right)^2$ — homogeneous. Put $y = vx$, $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$:

$v + x\dfrac{dv}{dx} = 1 + v + v^2 \Rightarrow x\dfrac{dv}{dx} = 1 + v^2$

Separate: $\dfrac{dv}{1+v^2} = \dfrac{dx}{x} \Rightarrow \tan^{-1} v = \log|x| + C$

General solution: $\tan^{-1}\dfrac{y}{x} = \log|x| + C$

Q29(b) Separate: $\dfrac{y}{y+2}\,dy = \dfrac{x+2}{x}\,dx \Rightarrow \left(1 - \dfrac{2}{y+2}\right)dy = \left(1 + \dfrac{2}{x}\right)dx$

Integrate: $y - 2\log|y+2| = x + 2\log|x| + C$

$y(1) = -1$: $-1 - 2\log 1 = 1 + 2\log 1 + C \Rightarrow C = -2$

Particular solution: $y - 2\log|y+2| = x + 2\log|x| - 2$

Q30 Draw $x + y = 7$ (through (7, 0), (0, 7)) and $2x - 3y = -6$ (through (0, 2), (–3, 0)). The origin satisfies both inequalities, so the feasible region is on the origin side of both lines, in the first quadrant (shaded).

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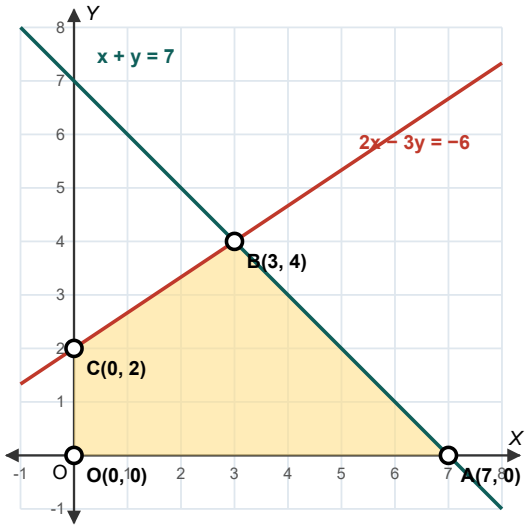

Corner points: O(0, 0), A(7, 0), B(3, 4) [solving $x + y = 7$ and $2x - 3y = -6$], C(0, 2).

Corner point	Z = 13x – 15y
O(0, 0)	0
A(7, 0)	91
B(3, 4)	39 – 60 = –21
C(0, 2)	–30

The region is bounded, so the minimum value is **Z = –30 at x = 0, y = 2**.

Q31(a) E_1: bag I chosen (die shows 1 or 2): $P(E_1) = \frac{2}{6} = \frac{1}{3}$; E_2: bag II: $P(E_2) = \frac{2}{3}$.

$P(R|E_1) = \frac{3}{7}$; $P(R|E_2) = \frac{8}{14} = \frac{4}{7}$.

By total probability: $P(R) = \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7} = \frac{1}{7} + \frac{8}{21} = \frac{11}{21}$

Q31(b) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = a$ ... (1)

P(exactly one) $= P(X) + P(Y) - 2P(X \cap Y) = b$ ... (2)

(1) – (2): $P(X \cap Y) = a - b$. From (1): $P(X) + P(Y) = a + (a - b) = 2a - b$.

$P(X') + P(Y') = [1 - P(X)] + [1 - P(Y)] = 2 - (2a - b) = 2 - 2a + b$. Hence proved.

Q32(a) **Reflexive:** for any $x \in Z$, $|x - x| = 0 = 0 \cdot p$ is divisible by p, so $(x, x) \in R$.

Symmetric: if $(x, y) \in R$, then $|x - y| = kp$ for some integer k. Since $|y - x| = |x - y| = kp$, $(y, x) \in R$.

Transitive: if $(x, y) \in R$ and $(y, z) \in R$, then $x - y = mp$ and $y - z = np$ for integers m, n (signs absorbed in m, n). Adding: $x - z = (m + n)p$, so $|x - z|$ is divisible by p and $(x, z) \in R$.

R is reflexive, symmetric and transitive, so **R is an equivalence relation**.

Q32(b) **One-one:** let $f(x_1) = f(x_2)$: $\frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3}$

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$(3x_1+2)(5x_2-3)=(3x_2+2)(5x_1-3)$

$15x_1x_2-9x_1+10x_2-6=15x_1x_2-9x_2+10x_1-6 \Rightarrow -19x_1=-19x_2 \Rightarrow x_1=x_2.$

So f is one-one.

Onto: let $y \in R-\left\{\frac{3}{5}\right\}$ and $f(x)=y$: $3x+2=5xy-3y \Rightarrow x(5y-3)=3y+2 \Rightarrow x=\frac{3y+2}{5y-3}$, which is a real number since $y \neq \frac{3}{5}$.

Also $x \neq \frac{3}{5}$: if $\frac{3y+2}{5y-3}=\frac{3}{5}$ then $15y+10=15y-9$, impossible. So $x \in R-\left\{\frac{3}{5}\right\}$ and $f(x)=y$; f is onto.

Hence f is one-one and onto.

Q33(a) $|A|=0(0-2)-2(0+2)+1(2+1)=-4+3=-1\neq 0$, so A^{-1} exists.

Cofactors: $C_{11}=-2, C_{12}=-2, C_{13}=3; C_{21}=-1, C_{22}=-1, C_{23}=2; C_{31}=-3, C_{32}=-2, C_{33}=4.$

$$\text{adj}A=\begin{bmatrix}-2&-1&-3\\-2&-1&-2\\3&2&4\end{bmatrix}, A^{-1}=\frac{1}{|A|}\text{adj}A=\begin{bmatrix}2&1&3\\2&1&2\\-3&-2&-4\end{bmatrix}$$

The system is $\begin{bmatrix}0&-2&1\\2&-1&-1\\1&-2&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\8\\10\end{bmatrix}$; the coefficient matrix is A'.

So $X=(A')^{-1}B=(A^{-1})'B=\begin{bmatrix}2&2&-3\\1&1&-2\\3&2&-4\end{bmatrix}\begin{bmatrix}7\\8\\10\end{bmatrix}=\begin{bmatrix}14+16-30\\7+8-20\\21+16-40\end{bmatrix}=\begin{bmatrix}0\\-5\\-3\end{bmatrix}$

x = 0, y = –5, z = –3 (check: $-2(-5)-3=7, 0+5+3=8, 0+10=10$ ✓)

Q33(b) Singular ⇒ determinant = 0. Expand along C_3:

$$\sin 3x\begin{vmatrix}-7&4\\-11&7\end{vmatrix}-\cos 2x\begin{vmatrix}3&-1\\-11&7\end{vmatrix}+2\begin{vmatrix}3&-1\\-7&4\end{vmatrix}=0$$

$\sin 3x(-49+44)-\cos 2x(21-11)+2(12-7)=0 \Rightarrow -5\sin 3x-10\cos 2x+10=0$

$\sin 3x+2\cos 2x-2=0 \Rightarrow (3\sin x-4\sin^3 x)+2(1-2\sin^2 x)-2=0$

$3\sin x-4\sin^3 x-4\sin^2 x=0 \Rightarrow \sin x(4\sin^2 x+4\sin x-3)=0 \Rightarrow \sin x(2\sin x-1)(2\sin x+3)=0$

$\sin x=0$ or $\sin x=\frac{1}{2}$ ($\sin x=-\frac{3}{2}$ is impossible).

In $\left[0,\frac{\pi}{2}\right]$: $x=0$ **or** $x=\frac{\pi}{6}$

Q34 $\frac{dx}{dt}=-\sin t, \frac{dy}{dt}=-m\sin mt$, so $\frac{dy}{dx}=\frac{m\sin mt}{\sin t}$.

$\sin t=\sqrt{1-x^2}$ and $\sin mt=\sqrt{1-y^2}$, so $\sqrt{1-x^2}\frac{dy}{dx}=m\sqrt{1-y^2}$.

Square: $(1-x^2)\left(\frac{dy}{dx}\right)^2=m^2(1-y^2)$

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Differentiate w.r.t. x: $(1-x^2)\cdot 2\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -2m^2y\frac{dy}{dx}$

Divide by $2\frac{dy}{dx}$: $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -m^2y$, i.e. $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$. Hence proved.

Q35

Direction ratios: (2, 3, 4) and (5, 2, 1). Since $\frac{2}{5} \neq \frac{3}{2}$, the lines are **not parallel**.

General points: L_1: $(2\lambda+1, 3\lambda+2, 4\lambda+3)$; L_2: $(5\mu+4, 2\mu+1, \mu)$.

Equate: $2\lambda+1 = 5\mu+4$ ... (1); $3\lambda+2 = 2\mu+1$ ... (2); $4\lambda+3 = \mu$ ... (3)

From (3) in (1): $2\lambda+1 = 20\lambda+15+4 \Rightarrow -18\lambda = 18 \Rightarrow \lambda = -1$, so $\mu = -1$.

Check (2): $3(-1)+2 = -1$ and $2(-1)+1 = -1$ ✓. The lines intersect.

Point of intersection: $\lambda = -1$ in L_1: **(–1, –1, –1)**

Q36

(i) Ten subscribers leave for every ₹ 1, so **10x** subscribers discontinue.

(ii) Subscribers $= 5000 - 10x$, fee $= 300 + x$: $R(x) = (300+x)(5000-10x)$

(iii) (a) $R'(x) = (5000-10x) - 10(300+x) = 2000 - 20x$; $R'(x) = 0 \Rightarrow x = 100$.

$R''(x) = -20 < 0$, so R is maximum at **x = 100** (fee ₹ 400).

OR (iii) (b) $R'(x) = 2000 - 20x > 0$ for $x < 100$ and < 0 for $x > 100$.

R(x) is **increasing in (0, 100)** and **decreasing in (100, 5000)**.

Q37

(i) **₹ 3,00,000, ₹ 2,00,000 or ₹ 50,000** (or nothing).

(ii) (a) Winning at least ₹ 2,00,000 means the first prize or a second prize: 1 + 2 = 3 tickets out of 1,00,000.

$$P = \frac{3}{100000}$$

OR (ii) (b) Winning tickets $= 1+2+3 = 6$. $P(\text{no prize}) = 1 - \frac{6}{100000} = \frac{99994}{100000} = \frac{49997}{50000}$

(iii) Let E_1: wins on the first ticket, $P(E_1) = \frac{6}{100000}$; E_2: wins on the second, $P(E_2) = \frac{1}{100000}$ (independent).

$P(\text{exactly one}) = P(E_1)P(E_2') + P(E_1')P(E_2) = \frac{6}{10^5}\cdot\frac{99999}{10^5} + \frac{99994}{10^5}\cdot\frac{1}{10^5} = \frac{699988}{10^{10}}$

Q38

(i) Two concentric circles with centre at the origin, radii 8 and 2:

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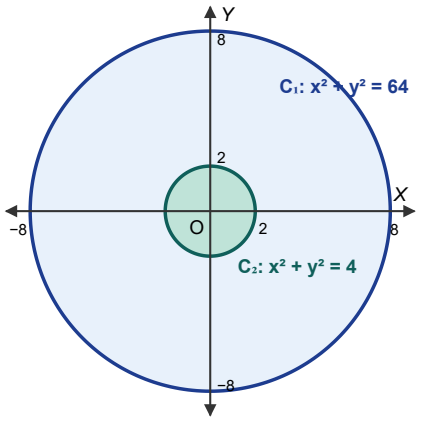

(ii) $C_1: y = \pm\sqrt{64 - x^2}$; $C_2: y = \pm\sqrt{4 - x^2}$.

(iii) (a) By symmetry, area $= 4\int_0^8 \sqrt{64 - x^2}\,dx = 4\left[\frac{x}{2}\sqrt{64 - x^2} + \frac{64}{2}\sin^{-1}\frac{x}{8}\right]_0^8$

$= 4\left[0 + 32 \cdot \frac{\pi}{2} - 0\right] = 64\pi$ **sq units**

OR (iii) (b) Area $= 4\int_0^2 \sqrt{4 - x^2}\,dx = 4\left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_0^2 = 4\left[2 \cdot \frac{\pi}{2}\right] = 4\pi$ **sq units**

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