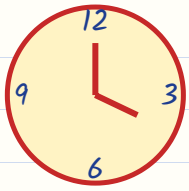
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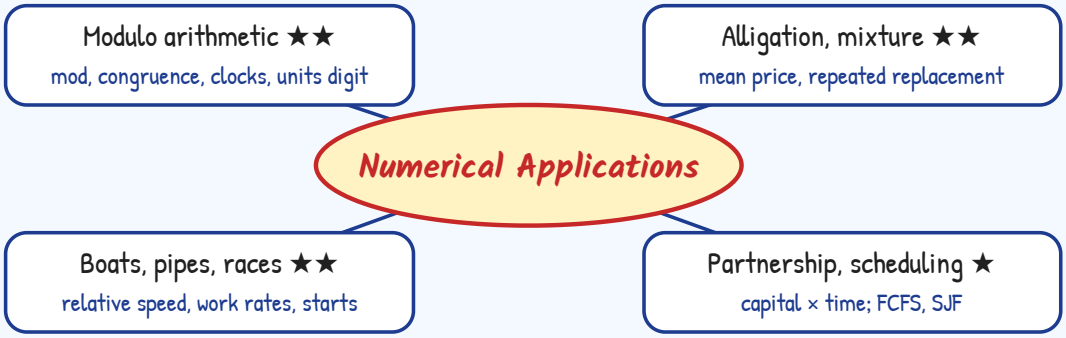
Chapter 1
Numbers, Quantification and Numerical Applications



clock = mod 12

CBSE Applied Mathematics (241) · Support material Unit 1 · CBSE board 2027

Chapter map (★ = asked often in CBSE papers 2024–2026)



What is inside

1. Concept notes with formulas and worked examples
2. CBSE support material exercises (all 60), fully solved
3. CBSE PYQs 2024–2026, fully solved
4. Quick-reference sheet; practice A–G, 20 MCQs, 6 A–R, answers, summary

Colour code used

red = key term green = definition

purple = sub-heading black = explanation

★ = high-frequency board topic

1 = mark the examiner gives for that point

★ **Exam tip:** Every board paper has 3–4 questions here worth 8–10 marks: a clock or mod MCQ, a units-digit or mod question, one mixture/boat/race problem and a 5-mark pipes-and-cisterns problem. Write the rate per hour first, then the equation.

Based on the CBSE Applied Mathematics Class XII support material (Unit 1) and the CBSE syllabus for 2026–27. All solutions written by us; where the book's answer key is wrong, the correct answer is given with a note. Board questions are quoted from the CBSE papers named in each tag.

1.1 Modulo arithmetic; 1.2 congruence modulo ★★

For integers a and $n > 0$, $a \bmod n$ is the remainder r when a is divided by n: $a = nq + r$ with $0 \leq r < n$. E.g. $25 \bmod 7 = 4$ since $25 = 7 \times 3 + 4$.

Negative numbers: the remainder must still be non-negative. $-41 \bmod 9$: $-41 = 9 \times (-5) + 4$, so the answer is 4 (not -5).

Rules (all results taken mod n)

operation	rule	example (mod 7)
addition	$(a+b) \bmod n = [(a \bmod n) + (b \bmod n)] \bmod n$	$(17+13) \bmod 7 = (3+6) \bmod 7 = 2$
subtraction	$(a-b) \bmod n = [(a \bmod n) - (b \bmod n)] \bmod n$	$(11-3) \bmod 7 = 1$
multiplication	$(ab) \bmod n = [(a \bmod n)(b \bmod n)] \bmod n$	$(16 \times 29) \bmod 7 = (2 \times 1) = 2$
powers	$a^k \bmod n = (a \bmod n)^k \bmod n$	$9^{20} \bmod 7 = 2^{20} \bmod 7$

Powers: find a small power that is $\equiv 1$ and use cycles. $3^6 \equiv 1 \pmod 7$, so $3^{128} = (3^6)^{21} \cdot 3^2 \equiv 9 \equiv 2$.

Clocks and calendars

A 12-hour clock works mod 12, a 24-hour day mod 24, days of the week mod 7. After 1275 hours from 9 p.m.: $1275 \bmod 24 = 3$, so 3 hours later: 12 midnight. After 200 hours from 5 a.m.: $200 = 24 \times 8 + 8$, so 1 p.m.

Units digit

The units digit of a^k is $a^k \bmod 10$; units digits repeat in cycles of 4 (or less). For 7: 7, 9, 3, 1. 7^{295}: $295 \bmod 4 = 3$, so units digit $= 3$.

last digit	cycle
2	2, 4, 8, 6
3	3, 9, 7, 1
7	7, 9, 3, 1
8	8, 4, 2, 6
4 / 9	4, 6 / 9, 1
0, 1, 5, 6	always the same

a is **congruent** to b modulo n, written $a \equiv b \pmod n$, if n divides $a - b$; i.e. a and b leave the same remainder on division by n. E.g. $38 \equiv 14 \pmod{12}$ since $12 \mid 24$.

Properties (for $a \equiv b, c \equiv d \pmod n$):

1. reflexive $a \equiv a$; symmetric $a \equiv b \Rightarrow b \equiv a$; transitive $a \equiv b,\ b \equiv c \Rightarrow a \equiv c$;
2. $a + c \equiv b + d,\ a - c \equiv b - d,\ ac \equiv bd \pmod n$;
3. $a^k \equiv b^k \pmod n$ for every positive integer k;
4. $a \equiv b \pmod n \iff a = b + kn$ for some integer k.

Equivalence (residue) classes

For modulo m there are exactly m classes $[0], [1], \ldots, [m-1]$; $[r] = \{a \in \mathbb{Z} : a \equiv r \pmod{m}\}$. E.g. mod 3: $[0] = \{\ldots, -3, 0, 3, 6, \ldots\}$, $[1] = \{\ldots, -2, 1, 4, 7, \ldots\}$, $[2] = \{\ldots, -1, 2, 5, 8, \ldots\}$. The classes do not overlap.

Solving $x \equiv a \pmod{n}$ **in a range:** list $a \bmod n$, then add n repeatedly. $x \equiv 23 \pmod{7}$ with $21 \le x < 31$: $23 \equiv 2$, so $x \in \{23, 30\}$.

Worked example

Remainder when $226 \times 369 \times 122 \times 461 \times 1025$ is divided by 8: remainders $2, 1, 2, 5, 1$; product $20 \equiv 4 \pmod{8}$. Remainder 4.

> ★ ***Exam tip:*** Board MCQs: "$-41 \bmod 9$" (answer 4), "time after 1275 hours", "number of articles left over". Always reduce each number first, then combine. In a 3-mark question on modular addition, show $17 +_{30} 13 = 0$ as $17 + 13 = 30 = 30 \times 1 + 0$.

1.3 Simple arithmetic functions; 1.4 alligation and mixture

★★

Arithmetic functions appear in the support material (Exercise 2) but are not in the 2026-27 CBSE syllabus list; learn them for completeness.

function	meaning	formula ($n = p_1^{a_1} p_2^{a_2} \cdots$)
$\varphi(n)$ Euler's totient	number of positive integers $\le n$ coprime to n	$n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots$; $\varphi(p) = p - 1$
$\tau(n)$	number of positive divisors	$(a_1 + 1)(a_2 + 1)\cdots$; $\tau(p) = 2$
$\sigma(n)$	sum of positive divisors	$\prod \frac{p_i^{a_i+1} - 1}{p_i - 1}$; $\sigma(p) = p + 1$
$\mu(n)$ Möbius	sign function on square-free numbers	$\mu(1) = 1$; 0 if a prime is repeated; $(-1)^k$ for k distinct primes

$\varphi, \tau, \sigma, \mu$ are **multiplicative**: $f(mn) = f(m)f(n)$ when $\gcd(m, n) = 1$. For a prime p: $\varphi(p) + \tau(p) = (p - 1) + 2 = p + 1 = \sigma(p)$.

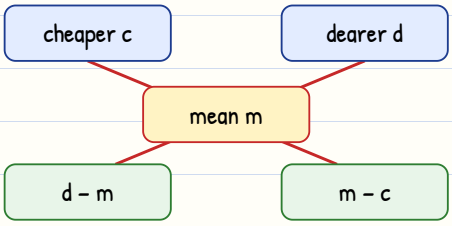
E.g. $72 = 2^3 3^2$: $\tau = 4 \times 3 = 12$, $\sigma = 15 \times 13 = 195$, $\varphi = 72 \cdot \frac{1}{2} \cdot \frac{2}{3} = 24$, $\mu(72) = 0$.

Alligation and mixture

Alligation is the rule for finding the ratio in which two (or more) ingredients at given prices must be mixed to get a mixture of a desired price (the **mean price**).

Rule of alligation (cheaper price c, dearer price d, mean price m, $c < m < d$):

$$\frac{\text{quantity of cheaper}}{\text{quantity of dearer}} = \frac{d - m}{m - c}$$



Cross the differences: cheaper : dearer = (d – m) : (m – c)

Mean price of a mixture of quantities q_1, q_2 at prices p_1, p_2: $m = \frac{p_1q_1+p_2q_2}{q_1+q_2}$ (weighted average). E.g. sugar at ₹ 25 and ₹ 35 in ratio 3 : 2: $m = \frac{75+70}{5} = ₹\,29$.

Same rule for percentages: gain %, concentration %, average salaries. E.g. 18% and 28% gain give 24% overall: $\frac{28-24}{24-18} = \frac{4}{6} = \frac{2}{3}$, so 2 : 3. Water has price 0 and concentration 0%.

Repeated replacement

A container has x units of liquid; y units are taken out and replaced by water, n times. Liquid left:

$$x\left(1-\frac{y}{x}\right)^n$$

E.g. 50 L of juice, 5 L replaced 5 times: $50(0.9)^5 = 29.52$ L.

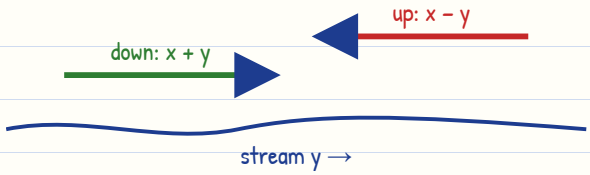
★ **Exam tip:** Check: the mean price must lie between the two prices; the ratio is "cheaper : dearer". For "how much to add" problems (jaggery syrup), use the concentration equation directly: $\frac{240+x}{600+x} = 0.5$.

I.5 Boats and streams; pipes and cisterns ★★

Boat speed in still water x km/h, stream speed y km/h:

direction	effective speed
downstream (with the stream)	$x + y$
upstream (against the stream)	$x - y$

If downstream speed $= a$ and upstream $= b$: boat $= \frac{a+b}{2}$, stream $= \frac{a-b}{2}$.



The stream helps one way and hinders the other

Round trip of distance d each way in total time T: $\frac{d}{x+y} + \frac{d}{x-y} = T$. With x unknown this gives a quadratic; reject the negative root.

Example (CBSE 2025): 3.5 km up and back in 1 h 12 min, stream 1 km/h: $\frac{3.5}{b-1} + \frac{3.5}{b+1} = 1.2 \Rightarrow 6b^2 - 35b - 6 = 0 \Rightarrow b = 6$ km/h.

Same time, different distances: $\frac{d_1}{x+y} = \frac{d_2}{x-y}$. "Takes thrice as long upstream" means $x + y = 3(x - y)$.

★ **Exam tip:** Convert minutes to hours first: 1 h 12 min $= \frac{6}{5}$ h, 4 h 16 min $= \frac{64}{15}$ h, 50 min $= \frac{5}{6}$ h.

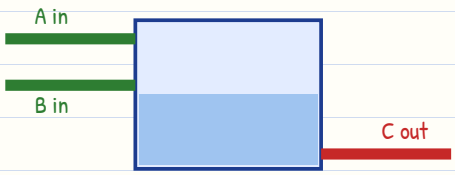
If a pipe fills a tank in n hours, it fills $\frac{1}{n}$ of the tank in 1 hour. Inlet pipes add, outlets (leaks) subtract:

situation	part filled per hour
A fills in a h, B in b h, together	$\frac{1}{a} + \frac{1}{b}$; time $= \frac{ab}{a+b}$
A fills in a h, C empties in c h ($c > a$)	$\frac{1}{a} - \frac{1}{c}$
a pipe fills in a h but with a leak in t h	leak empties in L: $\frac{1}{a} - \frac{1}{L} = \frac{1}{t}$

Pipes opened at different times: work in stages and add the parts done. In the CBSE 2024 cistern problem (A 3 h, B 4 h, C empties in 1 h; opened at 5, 6, 7 a.m.), the tank is $\frac{11}{12}$ full at 7 a.m. and then empties at $\frac{5}{12}$ per hour: empty after $\frac{11}{5}$ h, at 9:12 a.m.

A pipe closed after t minutes: (part by the pipe that ran throughout) + (part by the other in t) $= 1$. E.g. A 24 min, B 32 min, tank full in 18 min: $\frac{18}{24} + \frac{t}{32} = 1 \Rightarrow t = 8$.

Unknown single rates from combined rates: if $A+B+C$, $B+C$, $A+C$ are known, subtract: $A=(A+B+C)-(B+C)$ etc.



Net rate = sum of inlet rates − outlet rates

1.6 Races and games; partnership; scheduling ★

Races and games: the language

phrase	meaning
A beats B by x m	when A finishes the race of d m, B has run $d-x$ m
A beats B by t s	B finishes t s after A
A beats B by x m or t s	B runs those x m in t s: speed of B $=\frac{x}{t}$
A gives B a start of x m	B starts x m ahead; B runs $d-x$ while A runs d
dead heat	both reach the finishing post together

In the same time, distances are proportional to speeds: $\frac{d_A}{d_B}=\frac{v_A}{v_B}$.

Example: A beats B by 300 m and B beats C by 200 m in a 1000 m race. When A runs 1000, B runs 700, and C runs $700\times\frac{800}{1000}=560$: A beats C by 440 m.

Example (CBSE 2024): P beats Q by 18 m or 9 s in 1 km: Q's speed 2 m/s; Q takes 500 s; P takes $500-9=491$ s.

Partnership

Profit (or loss) is divided in the ratio of **capital × time** ("equivalent capital for 1 month"):

$$\text{A's profit} : \text{B's profit} = C_A t_A : C_B t_B$$

Simple partnership: same period, ratio of capitals. **Compound partnership**: different periods or changing capitals: add capital × months for each stage. A **working partner** first takes a salary (or commission); the rest is shared in the capital ratio.

Shared rent is split by months of occupancy per period: if 3 share for 4 months, 2 for 5 months and 1 for 3 months, each month's rent is divided among those present that month.

Scheduling (support material 1.5.5)

Not in the 2026-27 CBSE syllabus list; included because the support material has Exercise 8.

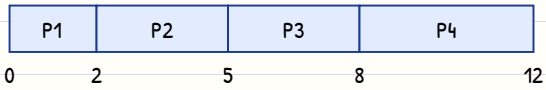
A **processor** executes **processes**; a **scheduling algorithm** decides the order.

term	formula
burst time (BT)	time a process needs on the CPU
completion time (CT)	time at which it finishes
turnaround time	TAT = CT − AT (arrival time)
waiting time	WT = TAT − BT = CT − AT − BT
idle time	CPU has nothing to run

FCFS (first come, first served): processes run in order of arrival. **SJF** (shortest job first, non-pre-emptive): when the CPU is free, the arrived process with the smallest burst time runs next. SJF gives the minimum average waiting time

but long jobs may **starve**.

Example: P1 (AT 0, BT 2), P2 (1, 3), P3 (5, 3), P4 (6, 4), SJF: Gantt P1 0–2, P2 2–5, P3 5–8, P4 8–12. TAT 2, 4, 3, 6 (avg 3.75); WT 0, 1, 0, 2 (avg 0.75).



Gantt chart for the SJF example

★ **Exam tip:** For every case question here, write: given data → what to find → formula in words → substitution → answer with units. Examiners give a mark for the setting-up equation even if arithmetic slips.

CBSE support material exercises, fully solved

Exercise 1 (modulo arithmetic)

Ex 1.1 Find the sum of 132 and 121 mod 23.

> **Ans.** $132 + 121 = 253 = 23 \times 11 + 0$, so $(132 + 121) \bmod 23 = 0$.

Ex 1.2 Find $7^6 \pmod 3$.

> **Ans.** $7 \equiv 1 \pmod 3$, so $7^6 \equiv 1^6 = 1 \pmod 3$.

Ex 1.3 Find x, given $x \equiv 23 \pmod 7$ and $21 \leq x < 31$.

> **Ans.** $23 = 7 \times 3 + 2$, so $x \equiv 2 \pmod 7$: $x = 2, 9, 16, 23, 30, \ldots$ In $21 \leq x < 31$: $x = 23$ or 30. (The book's key gives only 30; 23 also satisfies the condition.)

Ex 1.4 Find the remainder when $226 \times 369 \times 122 \times 461 \times 1025$ is divided by 8.

> **Ans.** Remainders mod 8: $226 \to 2, 369 \to 1, 122 \to 2, 461 \to 5, 1025 \to 1$. Product $2 \times 1 \times 2 \times 5 \times 1 = 20 \equiv 4 \pmod 8$. Remainder 4.

Ex 1.5 Evaluate $(16 \times 29) \pmod 7$.

> **Ans.** $16 \equiv 2, 29 \equiv 1 \pmod 7$; $(16 \times 29) \equiv 2 \times 1 = 2 \pmod 7$.

Ex 1.6 If $x \pmod 9 = 2$, find all possible x with $0 < x < 47$.

> **Ans.** $x = 9k + 2$: $2, 11, 20, 29, 38$ (47 is excluded). (The key omits 2, which also lies in $0 < x < 47$.)

Ex 1.7 Find the positive integers less than 50 forming the equivalence class 4 for modulo 6.

> **Ans.** $[4] = \{6k + 4\}$: $4, 10, 16, 22, 28, 34, 40, 46$.

Ex 1.8 What time will it be after 200 hours, if the present time is 5:00 a.m.?

> **Ans.** $200 = 24 \times 8 + 8$, so $200 \equiv 8 \pmod{24}$. 5 a.m. + 8 h = 1:00 p.m.

Ex 1.9 There are 81 boxes with 21 articles each. If the articles are rearranged with 5 in each box, how many are left without a box?

> **Ans.** Total $= 81 \times 21 = 1701$. $1701 \bmod 5 = 1$ (since $1701 = 5 \times 340 + 1$). 1 article is left.

Ex 1.10 Find $3^{128} \bmod 7$.

> **Ans.** $3^1 = 3,\ 3^2 = 2,\ 3^3 = 6,\ 3^6 \equiv 1 \pmod 7$. $128 = 6 \times 21 + 2$, so $3^{128} = (3^6)^{21} \cdot 3^2 \equiv 1 \cdot 9 \equiv 2 \pmod 7$.

Exercise 2 (arithmetic functions)

Ex 2.1 Find $\varphi(35)$.

> **Ans.** $35 = 5 \times 7$: $\varphi(35) = \varphi(5)\varphi(7) = 4 \times 6 = 24$.

Ex 2.2 Find $\mu(75) + \mu(85)$.

Ans. $75 = 3 \times 5^2$ (a repeated prime): $\mu(75) = 0$. $85 = 5 \times 17$ (two distinct primes): $\mu(85) = (-1)^2 = 1$. Sum $= 1$.

Ex 2.3 Calculate $\varphi(90) + \tau(42) + \sigma(72) + \mu(70)$.

Ans. $\varphi(90) = 90 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{4}{5} = 24$; $\tau(42)$: $42 = 2 \cdot 3 \cdot 7$, $2 \times 2 \times 2 = 8$; $\sigma(72)$: $72 = 2^3 3^2$, $(1 + 2 + 4 + 8)(1 + 3 + 9) = 195$; $\mu(70)$: $70 = 2 \cdot 5 \cdot 7$, $(-1)^3 = -1$.
Sum $= 24 + 8 + 195 - 1 = 226$.

Ex 2.4 Verify whether $\varphi(p) + \tau(p) = \sigma(p)$ holds for $p = 24$.

Ans. $24 = 2^3 \cdot 3$: $\varphi(24) = 24 \cdot \frac{1}{2} \cdot \frac{2}{3} = 8$, $\tau(24) = 4 \times 2 = 8$, $\sigma(24) = 15 \times 4 = 60$. $\varphi + \tau = 16 \neq 60$: the relation does **not** hold for 24; it holds only for primes ($p - 1 + 2 = p + 1$).

Ex 2.5 For prime p, $\varphi(p) + \tau(p) = \sigma(p)$. Complete the table for $p = 2, 3, 5, 7, 11$.

Ans.

p	$\varphi = p - 1$	$\tau = 2$	$\sigma = p + 1$	**check**
2	1	2	3	3 yes
3	2	2	4	4 yes
5	4	2	6	6 yes
7	6	2	8	8 yes
11	10	2	12	12 yes

Ex 2.6 If $n = 2p$ with p an odd prime less than 14, (i) tabulate $\varphi(n), \tau(n), \sigma(n)$; (ii) verify $n + \varphi(n) + \tau(n) = \sigma(n)$.

Ans. $\varphi(2p) = p - 1, \tau(2p) = 4, \sigma(2p) = 3(p + 1)$.

p	n	φ	τ	σ	$n + \varphi + \tau$
3	6	2	4	12	12
5	10	4	4	18	18
7	14	6	4	24	24
11	22	10	4	36	36
13	26	12	4	42	42

In general $2p + (p - 1) + 4 = 3p + 3 = \sigma(n)$: verified.

Ex 2.7 Show that $\tau(n)$ and $\sigma(n)$ are multiplicative for $n = 24$.

Ans. $24 = 3 \times 8$ with $\gcd(3, 8) = 1$. $\tau(24) = 8$ and $\tau(3)\tau(8) = 2 \times 4 = 8$. $\sigma(24) = 60$ and $\sigma(3)\sigma(8) = 4 \times 15 = 60$. Hence $f(24) = f(3)f(8)$ for both.

Exercise 3 (alligation and mixture)

Ex 3.1 In what ratio must rice at ₹ 69/kg be mixed with rice at ₹ 100/kg so that the mixture is worth ₹ 80/kg?

Ans. Cheaper : dearer $= (100 - 80) : (80 - 69) = 20 : 11$.

Ex 3.2 The average salary of all staff (supervisors and labourers) is ₹ 5,750; that of a supervisor ₹ 20,000 and of a labourer ₹ 5,000. Find the number of labourers if there are 4 supervisors.

Ans. Let there be n labourers: $4 \times 20000 + 5000n = 5750(n + 4) \Rightarrow 80000 + 5000n = 5750n + 23000 \Rightarrow 750n = 57000 \Rightarrow n = 76$.

(By alligation: labourers : supervisors $= (20000-5750) : (5750-5000) = 14250 : 750 = 19 : 1$, so $19 \times 4 = 76$.)

Ex 3.3 A container has 70 L of orange squash; 7 L is taken out and replaced by water, and this is done thrice. How much squash is left?

Ans. Squash left $= 70\left(1-\frac{7}{70}\right)^3 = 70 \times (0.9)^3 = 70 \times 0.729 = 51.03$ L.

Ex 3.4 Pulses at ₹ 55/kg and ₹ 90/kg are mixed in the ratio 2 : 3. Find the price of the mixture per kg.

Ans. $m = \frac{2\times55+3\times90}{5} = \frac{110+270}{5} =$ ₹ 76 per kg.

Ex 3.5 A shopkeeper has 1 quintal of wheat; he sells part at 18% gain and the rest at 28% gain, gaining 24% overall. Find the quantity sold at each rate.

Ans. Ratio (18% part : 28% part) $= (28-24) : (24-18) = 4 : 6 = 2 : 3$. 1 quintal $= 100$ kg: at 18%: $\frac{2}{5} \times 100 = 40$ kg; at 28%: 60 kg. (The book's key prints 400 kg and 600 kg, which would total 10 quintals; 40 kg and 60 kg is correct.)

Ex 3.6 600 g of jaggery syrup has 40% jaggery. How much jaggery should be added to make it 50%?

Ans. Jaggery $= 40\%$ of $600 = 240$ g. Adding x g: $\frac{240+x}{600+x} = \frac{1}{2} \Rightarrow 480 + 2x = 600 + x \Rightarrow x = 120$ g.

Ex 3.7 In what ratio must water be added to honey costing ₹ 240 per litre so that the syrup is worth ₹ 200 per litre?

Ans. Water costs ₹ 0. Water : honey $= (240-200) : (200-0) = 40 : 200 = 1 : 5$.

Ex 3.8 A container has 50 L of juice; 5 L is taken out and replaced by water, and this is repeated 4 more times. Find the juice left.

Ans. 5 operations in all: $50(1-0.1)^5 = 50 \times 0.59049 = 29.52$ L.

Exercise 4 (boats and streams)

Ex 4.1 Find the speed of the boat if it moves downstream at 16 km/h and upstream at 10 km/h.

Ans. Boat $= \frac{16+10}{2} = 13$ km/h (stream $= \frac{16-10}{2} = 3$ km/h).

Ex 4.2 A boat's still-water speed is 14 km/h; downstream it moves at 24 km/h. Find its speed against the stream.

Ans. Stream $= 24 - 14 = 10$ km/h; upstream $= 14 - 10 = 4$ km/h.

Ex 4.3 A boat goes 8 km upstream and returns; the total time is 4 h 16 min. If the current is 1 km/h, find the actual speed of the boat.

Ans. 4 h 16 min $= \frac{64}{15}$ h. $\frac{8}{b-1} + \frac{8}{b+1} = \frac{64}{15} \Rightarrow \frac{16b}{b^2-1} = \frac{64}{15} \Rightarrow 4b^2 - 15b - 4 = 0 \Rightarrow (4b+1)(b-4) = 0$. $b = 4$ km/h.

Ex 4.4 A man rows at 7 km/h in still water; the stream flows at 5 km/h. It takes him 7 hours to go to a place and return. How far is the place?

Ans. Downstream 12 km/h, upstream 2 km/h: $\frac{d}{12} + \frac{d}{2} = 7 \Rightarrow \frac{7d}{12} = 7 \Rightarrow d = 12$ km.

Ex 4.5 A boat covers 32 km upstream and 36 km downstream in 7 hours, and 40 km upstream and 48 km downstream in 9 hours. Find the speeds of the boat and the stream.

Ans. Let $\frac{1}{u} = a$ (upstream), $\frac{1}{d} = b$ (downstream): $32a + 36b = 7$, $40a + 48b = 9$. $\times 4$ and $\times 3$: $128a + 144b = 28$, $120a + 144b = 27 \Rightarrow 8a = 1$, $a = \frac{1}{8}$; then $36b = 3$, $b = \frac{1}{12}$. Upstream 8 km/h, downstream 12 km/h: boat $= 10$ km/h, stream $= 2$ km/h.

Ex 4.6 A man rows at $7\frac{1}{2}$ km/h in still water; the river flows at 1.5 km/h. He takes 50 minutes to go to a place and back. How far is it?

Ans. Downstream 9, upstream 6 km/h: $\frac{d}{9} + \frac{d}{6} = \frac{5}{6} \Rightarrow \frac{5d}{18} = \frac{5}{6} \Rightarrow d = 3$ km.

Ex 4.7 Speeds of a motor boat and the current are in ratio 36 : 5. The boat goes downstream in 5 h 10 min. How long will it take to come back?

> **Ans.** Downstream $41k$, upstream $31k$. Time is inversely proportional to speed: $\frac{31}{6} \times \frac{41}{31} = \frac{41}{6}$ h $= 6\frac{5}{6}$ h $= 6$ h 50 min.

Exercise 5 (pipes and cisterns)

Ex 5.1 Pipe A fills a tank in 30 h and B in 45 h. How long will both take?

> **Ans.** $\frac{1}{30} + \frac{1}{45} = \frac{3+2}{90} = \frac{1}{18}$ per hour: 18 hours.

Ex 5.2 A pipe fills a cistern in 6 h; with a leak it takes 9 h. How long will the leak take to empty the full tank?

> **Ans.** $\frac{1}{6} - \frac{1}{L} = \frac{1}{9} \Rightarrow \frac{1}{L} = \frac{1}{6} - \frac{1}{9} = \frac{1}{18}$: 18 hours.

Ex 5.3 Pipes A and B fill a cistern in 4 h and 6 h; pipe C empties it in 8 h. If all are opened together, when will it be filled?

> **Ans.** $\frac{1}{4} + \frac{1}{6} - \frac{1}{8} = \frac{6+4-3}{24} = \frac{7}{24}$: time $= \frac{24}{7}$ h $= 3$ h $25\frac{5}{7}$ min ($\approx$ 3 h 26 min).

Ex 5.4 A cistern fills in 8 h, but because of a leak it takes 2 hours more. How long will the leak take to empty the full cistern?

> **Ans.** $\frac{1}{8} - \frac{1}{L} = \frac{1}{10} \Rightarrow \frac{1}{L} = \frac{1}{40}$: 40 hours.

Ex 5.5 An inlet fills a cistern in 20 h and an outlet empties it in 25 h. Both are opened; after 10 h the outlet is closed. Find the total time to fill.

> **Ans.** In 10 h: $10\left(\frac{1}{20} - \frac{1}{25}\right) = 10 \times \frac{1}{100} = \frac{1}{10}$. Remaining $\frac{9}{10}$ by the inlet: $\frac{9}{10} \times 20 = 18$ h. Total $= 28$ hours.

Ex 5.6 Pipes A and B fill a tank in 24 and 32 minutes. Both are opened; after how long should B be closed so that the tank is full in 18 minutes?

> **Ans.** A runs 18 min: $\frac{18}{24} = \frac{3}{4}$. B must do $\frac{1}{4}$: $\frac{t}{32} = \frac{1}{4} \Rightarrow t = 8$ minutes.

Ex 5.7 Taps A, B, C together drain a tank in $1\frac{1}{2}$ min; B and C together in 2 min; A and C in $2\frac{4}{13}$ min. How long will A and B together take?

> **Ans.** Rates per minute: $A + B + C = \frac{2}{3}$, $B + C = \frac{1}{2}$, $A + C = \frac{13}{30}$. $A = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$; $B = \frac{2}{3} - \frac{13}{30} = \frac{7}{30}$. $A + B = \frac{5}{30} + \frac{7}{30} = \frac{2}{5}$: time $= \frac{5}{2}$ min.

Exercise 6 (races and games)

Ex 6.1 In a 1000 m race A defeats B by 300 m and B defeats C by 200 m. By how much will A defeat C?

> **Ans.** A 1000 : B 700; B 1000 : C 800, so C runs $700 \times 0.8 = 560$ when A finishes. A beats C by 440 m.

Ex 6.2 In a 1000 m race A gives B a start of 100 m and C a start of 280 m. How much start can B give C?

> **Ans.** A 1000 : B 900 : C 720. When B runs 1000, C runs $720 \times \frac{1000}{900} = 800$. B can give C 200 m.

Ex 6.3 In a 500 m race A defeats B by 60 m or 12 s. Find A's time.

> **Ans.** B runs 60 m in 12 s: 5 m/s; B's time $= \frac{500}{5} = 100$ s; A's time $= 100 - 12 = 88$ s.

Ex 6.4 In a 900 m race A gives B a start of 150 m and defeats him by 50 s. If A's speed is 4.5 m/s, find B's speed.

> **Ans.** A's time $= \frac{900}{4.5} = 200$ s. B runs $900 - 150 = 750$ m in $200 + 50 = 250$ s: 3 m/s.

Ex 6.5 A runs 3 times as fast as B. If A gives B a start of 40 m, how far must the goal be so that they finish together?

> **Ans.** $\frac{d}{d-40} = 3 \Rightarrow d = 3d - 120 \Rightarrow d = 60$ m.

Ex 6.6 A team played 40 games and lost 16. What percent did it win?

> **Ans.** Won $= 24$; $\frac{24}{40} \times 100 = 60\%$.

Ex 6.7 In a 200 m race Prateek beats Samarth by 35 m or 7 s. How long did Prateek take?

> **Ans.** Samarth: 35 m in 7 s, 5 m/s; his time $= 40$ s; Prateek $= 40 - 7 = 33$ s.

Exercise 7 (partnership)

Ex 7.1 B contributes $\frac{1}{3}$ of the capital and A as much as B and C together. Find the ratio of capitals.

> **Ans.** A $= \frac{1}{2}$ of the total (A = B + C and A + B + C = 1). C $= 1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$. A : B : C $= \frac{1}{2} : \frac{1}{3} : \frac{1}{6} = 3 : 2 : 1$.

Ex 7.2 Rahul got ₹ 5,000 of a total profit of ₹ 9,000. Ramesh invested ₹ 3,000 for 6 months; Rahul invested for the whole year. Find Rahul's investment.

> **Ans.** Ramesh's share $= 4000$. $\frac{3000\times6}{x\times12} = \frac{4000}{5000} \Rightarrow \frac{18000}{12x} = \frac{4}{5} \Rightarrow x = ₹\,1{,}875$.

Ex 7.3 Priya invested ₹ 40,000; after 4 months Rekha invested ₹ 50,000. Find Rekha's share of a yearly profit of ₹ 2,20,000.

> **Ans.** $40000 \times 12 : 50000 \times 8 = 480000 : 400000 = 6 : 5$. Rekha $= \frac{5}{11} \times 220000 = ₹\,1{,}00{,}000$.

Ex 7.4 Profits of A, B, C are in ratio 6 : 7 : 8 and their investment periods 2 : 3 : 4. Find the investment ratio.

> **Ans.** Investment $\propto \frac{\text{profit}}{\text{time}}$: $\frac{6}{2} : \frac{7}{3} : \frac{8}{4} = 3 : \frac{7}{3} : 2 = 9 : 7 : 6$.

Ex 7.5 A and B invest ₹ 30,000 and ₹ 60,000. From a yearly profit of ₹ 18,000, A gets ₹ 8,500 as his profit share plus salary for managing. Find A's salary.

> **Ans.** Let the salary be S; the rest $18000 - S$ is shared 1 : 2. $S + \frac{18000-S}{3} = 8500 \Rightarrow 3S + 18000 - S = 25500 \Rightarrow S = ₹\,3{,}750$.
>
> Check: $3750 + \frac{14250}{3} = 3750 + 4750 = 8500$. (The book's key gives ₹ 2,500, which gives A only ₹ 7,666.67.)

Ex 7.6 Prakash, Sachin and Anil invested ₹ 11, 16.5 and 8.25 lakh; the 3-year profit is ₹ 19.5 lakh. Find 50% of Anil's share.

> **Ans.** Ratio $11 : 16.5 : 8.25 = 4 : 6 : 3$ ($\times\frac{4}{11}$). Anil $= \frac{3}{13} \times 19.5 = 4.5$ lakh; 50% $= ₹\,2.25$ lakh.

Ex 7.7 Nikhil, Priyesh and Ritik rented a house for ₹ 13,824 a year. All three stayed 4 months, then Ritik left; after 5 more months Priyesh left. How much should each pay?

> **Ans.** Monthly rent $= \frac{13824}{12} = 1152$. First 4 months shared by 3: $\frac{4\times1152}{3} = 1536$ each; next 5 months by 2: $\frac{5\times1152}{2} = 2880$ each; last 3 months Nikhil alone: 3456.
>
> Ritik ₹ 1,536; Priyesh $1536 + 2880 = ₹\,4{,}416$; Nikhil $1536 + 2880 + 3456 = ₹\,7{,}872$ (total 13,824).

Ex 7.8 Arun invested ₹ 38,000; after 5 months Bakul joined with ₹ 55,000. The year's profit is ₹ 22,000. Find the approximate difference between their shares.

> **Ans.** $38000 \times 12 : 55000 \times 7 = 456 : 385$ (total 841). Arun $= 22000 \times \frac{456}{841} = 11928.66$; Bakul $= 10071.34$. Difference ≈ ₹ 1,857.

Exercise 8 (scheduling)

Ex 8.1 Which is false about SJF? S1: it causes minimum average waiting time; S2: it can cause starvation. (A) only S1 (B) only S2 (C) both (D) neither

> **Ans.** Both statements are true, so neither is false. **(D)**

Ex 8.2 State the relationship between arrival, burst, completion and waiting time.

> **Ans.** Waiting time = completion time − arrival time − burst time (WT = TAT − BT, TAT = CT − AT).

Ex 8.3 P1–P4 arrive at 0 with burst times 20, 5, 8, 44. Find the average waiting time by FCFS.

> **Ans.** Order P1, P2, P3, P4: waits 0, 20, 25, 33. Average $= \frac{78}{4} = 19.5$ units.

Ex 8.4 Arrival 0 for all; burst times P1 20, P2 5, P3 9, P4 4. Average waiting time by SJF?

> **Ans.** Order P4 (4), P2 (5), P3 (9), P1 (20): waits 0, 4, 9, 18. Average $= \frac{31}{4} = 7.75$ units.

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Ex 8.5 P1–P4 arrive at 1, 2, 3, 4 with bursts 3, 4, 2, 4. (a) When are all executed? (b) Average turnaround time by non-pre-emptive FCFS.

> **Ans.** Idle 0–1; P1 1–4, P2 4–8, P3 8–10, P4 10–14. (a) 14 units. (b) TAT $= 3, 6, 7, 10$: average $\frac{26}{4} = 6.5$ units (average waiting $= \frac{0+2+5+6}{4} = 3.25$).

Ex 8.6 P1–P4 arrive at 0, 1, 5, 6 with bursts 2, 3, 3, 4. (a) When are all executed? (b) Average waiting and turnaround time by non-pre-emptive SJF.

> **Ans.** P1 0–2, P2 2–5, P3 5–8, P4 8–12. (a) 12 units. (b) TAT $2, 4, 3, 6$: average 3.75; WT $0, 1, 0, 2$: average 0.75 units.

Quick-reference sheet

topic	formula / rule
$a \bmod n$	remainder r, $0 \le r < n$, in $a = nq + r$
$a \equiv b \pmod n$	$n \mid (a - b)$
clocks	hours mod 24 (or 12); days mod 7
units digit of 7^k	cycle 7, 9, 3, 1 (use $k \bmod 4$)
alligation	cheaper : dearer $= (d - m) : (m - c)$
replacement	$x(1 - \frac{y}{x})^n$
boats	down $x + y$, up $x - y$; boat $\frac{a+b}{2}$, stream $\frac{a-b}{2}$
pipes	rate $\frac{1}{n}$ per hour; outlets negative
races	same time: distance $\propto$ speed; "by x m or t s" gives loser's speed $\frac{x}{t}$
partnership	profit $\propto$ capital $\times$ time; salary first
scheduling	TAT = CT − AT; WT = TAT − BT

Case study (practice): 'The water park'. A water park has a pool filled by pipe A in 6 h and pipe B in 9 h, and drained by outlet C in 12 h. On Monday A and B were opened at 6 a.m., and C was opened by mistake at 7 a.m.
(i) What part of the pool was filled by 7 a.m.? (ii) What is the net filling rate after 7 a.m.? (iii) At what time will the pool be full? (iv) If instead only A and C are open, how long will it take to fill the pool?

★ **Exam tip:** Write the rates as fractions of the tank per hour and keep them as fractions until the end; decimals cause rounding slips.

CBSE board questions 2024–2026, fully solved

Every question on this chapter from the CBSE Applied Mathematics (241) board papers of 2024, 2025 and 2026 (paper 465), quoted with complete solutions. "(OR)" items are the internal choice of the question above them.

A. MCQs and Assertion–Reason (1 mark each)

CBSE 2024 · 465 In a 1 km race, player P beats player Q by 18 metres or 9 seconds. What is P's time to complete the race? (A) 512 s (B) 502 s (C) 491 s (D) 481 s [1]

> **Ans.** Q covers 18 m in 9 s, so Q's speed $= 2$ m/s and Q takes $\frac{1000}{2} = 500$ s. P finishes 9 s earlier: $500 - 9 = 491$ s. **(C)**

CBSE 2024 · 465 What time will it be after 1275 hours, if the present time is 9:00 p.m.? (A) 11 p.m. (B) 12 p.m. (C) 9 p.m. (D) 9 a.m. [1]

Ans. Working in mod 24: $1275 = 24 \times 53 + 3$, so $1275 \equiv 3 \pmod{24}$. 9 p.m. + 3 hours = 12 midnight (printed as 12 p.m.). **(B)**

CBSE 2025 · 465 $-41 \bmod 9$ is: (A) 5 (B) 4 (C) 3 (D) 0 [1]

Ans. $-41 = 9 \times (-5) + 4$ with $0 \le 4 < 9$, so $-41 \bmod 9 = 4$. **(B)**

B. Very short answer (2 marks)

CBSE 2025 · 465 Type I sugar costs ₹ 25 per kg and Type II ₹ 35 per kg. If they are mixed in the ratio 3 : 2, find the price per kg of the mixture. [2]

Ans. Mean price $= \frac{3\times25+2\times35}{3+2} = \frac{75+70}{5} =$ ₹ 29 per kg.

CBSE 2025 · 465 (OR) Pipe A can fill a tank in 1 hour and pipe B in $1\frac{1}{2}$ hours. If both are opened in the empty tank, how much time will they take to fill it? [2]

Ans. Part filled per hour: A $= 1$, B $= \frac{2}{3}$; together $= \frac{5}{3}$. Time $= \frac{3}{5}$ h $= 36$ minutes.

CBSE 2025 · 465 A boat goes 3.5 km upstream and returns, taking 1 h 12 min in all. If the current is 1 km/h, find the speed of the boat in still water. [2]

Ans. Let the speed be b km/h. Total time $\frac{3.5}{b-1} + \frac{3.5}{b+1} = \frac{12}{10}$ h.

$\frac{3.5\times2b}{b^2-1} = 1.2 \Rightarrow 7b = 1.2b^2 - 1.2 \Rightarrow 6b^2 - 35b - 6 = 0 \Rightarrow (6b+1)(b-6) = 0.$

$b = 6$ km/h (speed cannot be negative).

CBSE 2025 · 465 A runs $\frac{3}{2}$ times as fast as B. If A gives B a start of 40 m, how far must the winning post be so that A and B reach it at the same time? [2]

Ans. Let the race be d m. In the same time A runs d and B runs $d - 40$; distances are in the ratio of speeds:

$\frac{d}{d-40} = \frac{3}{2} \Rightarrow 2d = 3d - 120 \Rightarrow d = 120$ m.

CBSE 2026 · 465 A man goes 12 km downstream and comes back, rowing non-stop, in a total of 3 hours. If the stream flows at 3 km/h, find his speed in still water. [2]

Ans. Let the speed be u km/h: $\frac{12}{u+3} + \frac{12}{u-3} = 3 \Rightarrow \frac{24u}{u^2-9} = 3 \Rightarrow u^2 - 8u - 9 = 0 \Rightarrow (u-9)(u+1) = 0.$

$u = 9$ km/h.

C. Short answer (3 marks)

CBSE 2024 · 465 A container has 50 litres of juice. 5 litres are taken out and replaced by 5 litres of water; this is repeated 4 more times. Find the quantity of juice after the final replacement. [Use $(0.9)^5 = 0.59049$] [3]

Ans. Each operation leaves $\left(1 - \frac{5}{50}\right) = 0.9$ of the juice. After 5 operations: juice $= 50 \times (0.9)^5 = 50 \times 0.59049 = 29.5245$ litres.

CBSE 2025 · 465 Find the units digit in 7^{295}. [3]

Ans. Units digits of powers of 7: $7^1 \to 7$, $7^2 \to 9$, $7^3 \to 3$, $7^4 \to 1$, then repeat (cycle of 4).

$295 = 4 \times 73 + 3$, so 7^{295} has the same units digit as $7^3 = 343$.

Units digit $= 3$.

CBSE 2026 · 465 (i) Apply addition modulo to 17 and 13 for modulo 30. (ii) Find subtraction modulo 8 for 11 and 3. [3]

> **Ans.** (i) $17 + 13 = 30 = 30 \times 1 + 0$, so $17 +_{30} 13 = 0$.
>
> (ii) $11 - 3 = 8 = 8 \times 1 + 0$, so $11 -_8 3 = 0$.

CBSE 2026 · 465 (OR) Pipes A, B and C together fill a tank in 8 hours. After 2 hours together, B is closed and A and C fill the rest in 9 hours. Find the time in which B alone can fill the tank. [3]

> **Ans.** A + B + C fill $\frac{1}{8}$ per hour; in 2 hours $\frac{1}{4}$, leaving $\frac{3}{4}$.
>
> A + C fill $\frac{3}{4}$ in 9 hours: rate $\frac{3/4}{9} = \frac{1}{12}$ per hour.
>
> B alone: $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$ per hour, so B fills the tank in **24 hours**.

D. Long answer (5 marks)

CBSE 2024 · 465 A cistern has inlet pipes A and B and outlet pipe C. A and B can fill it separately in 3 h and 4 h; C can empty the full cistern in 1 h. If A, B and C are opened at 5, 6 and 7 a.m. respectively, at what time will the cistern be empty? [5]

> **Ans.** Work per hour: A $= \frac{1}{3}$, B $= \frac{1}{4}$, C $= -1$.
>
> 5 to 6 a.m. (A alone): $\frac{1}{3}$ filled.
>
> 6 to 7 a.m. (A + B): $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$; total $= \frac{1}{3} + \frac{7}{12} = \frac{11}{12}$.
>
> From 7 a.m. all three: net rate $= \frac{1}{3} + \frac{1}{4} - 1 = -\frac{5}{12}$ per hour (emptying).
>
> Time to empty $\frac{11}{12}$: $\frac{11/12}{5/12} = \frac{11}{5}$ h $= 2$ h 12 min.
>
> The cistern will be empty at **9:12 a.m.**

What the board asked (3 papers, 2024–2026)

Type	2024	2025	2026	Total
MCQs and Assertion–Reason (1 mark each)	2	1	0	3
Very short answer (2 marks)	0	4	1	5
Short answer (3 marks)	1	1	2	4
Long answer (5 marks)	1	0	0	1

Practice set: test yourself *Exam-style*

Try every question before looking at the answers at the end of the chapter. 56 or more out of 70 means you are ready for the board.

A. Fill in the blanks (1 mark each)

1. $47 \bmod 5 =$ _____.
2. $-17 \bmod 6 =$ _____.
3. The number of equivalence classes modulo 9 is _____.
4. The units digit of 3^{50} is _____.
5. If downstream speed is 18 and upstream 12 km/h, the stream speed is _____ km/h.
6. Water mixed with milk has price ₹ _____ per litre.
7. Profit in partnership is shared in the ratio of capital × _____.
8. Waiting time = turnaround time − _____ time.

B. True or false? (1 mark each)

1. $38 \equiv 14 \pmod{12}$.
2. $a \bmod n$ can be negative.
3. The mean price lies between the two prices.
4. Upstream speed is more than downstream speed.
5. A leak has a negative rate.
6. "A gives B a start of 20 m" means B starts 20 m behind.
7. $\varphi(p) = p - 1$ for a prime p.
8. SJF can cause starvation.

C. Very short answer (1 mark each)

1. Find $(23 \times 19) \bmod 5$.
2. What time is 50 hours after 10 a.m.?
3. Find $5^{40} \bmod 6$.
4. Find the ratio to mix tea at ₹ 120 and ₹ 180 for ₹ 160.
5. A boat goes 20 km downstream in 2 h. Find the downstream speed.
6. A tap fills a tank in 5 h. What part does it fill in 1 h?
7. A runs 1000 m in 200 s. Find his speed.
8. Write the formula for repeated replacement.
9. Find $\tau(36)$.
10. What is turnaround time?

D. Short answer I (2 marks each)

1. Find the remainder when 2^{100} is divided by 3.
2. Find the units digit of 8^{123}.
3. Coffee at ₹ 200/kg and ₹ 260/kg is mixed 2 : 1. Find the price of the mixture.
4. A boat goes 24 km downstream in 3 h and returns in 4 h. Find the speeds.
5. Pipes A and B fill a tank in 10 h and 15 h. Find the time together.
6. In a 100 m race A beats B by 10 m. B's speed is 9 m/s. Find A's time.
7. A and B invest ₹ 4,000 for 6 months and ₹ 3,000 for 12 months. Divide a profit of ₹ 5,000.
8. List all x with $x \equiv 3 \pmod{5}$ and $10 < x < 30$.

My score (fill in after checking the answers)

Section	A /8	B /8	C /10	D /16	E /18	F /10	G (bonus)	Total /70
My marks								

E. Short answer II (3 marks each)

1. Find the units digit of $17^{43} \times 23^{22}$.
2. A 60 L mixture has milk and water in ratio 2 : 1. How much water must be added to make the ratio 1 : 1?

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3. A man rows 30 km downstream and 18 km upstream, taking 3 h each time. Find his still-water speed and the stream's speed.
4. A cistern has two inlets filling it in 12 h and 15 h and an outlet emptying it in 20 h. All open together: when is it full?
5. In a 1 km race A beats B by 40 m and B beats C by 50 m. By how much does A beat C?
6. A, B, C start a business with ₹ 20,000, ₹ 30,000, ₹ 40,000; after 6 months A doubles his capital. Divide a year's profit of ₹ 27,000.

F. Long answer (5 marks each)

1. Three taps A, B, C fill a tank in 6, 8 and 12 hours. A and B are opened at 8 a.m., C at 9 a.m. When will the tank be full?
2. A container has 80 L of milk. 8 L is taken out and replaced by water, 3 times. Find the milk left, and the ratio of milk to water.

G. HOTS (higher-order thinking)

1. Why does the remainder in mod n never reach n?
2. Can the mean price of a mixture be outside the two given prices? Why?
3. Why does the round trip in a stream always take longer than in still water?
4. Why is the rent problem (Ex 7.7) not split in the simple ratio 12 : 9 : 4?

20 MCQs

1. $58 \bmod 7$ is:
   (a) 1 (b) 3 (c) 2 (d) 4
2. $-23 \bmod 5$ is:
   (a) 2 (b) -3 (c) 3 (d) 4
3. $(15 \times 17) \bmod 4$ is:
   (a) 1 (b) 0 (c) 2 (d) 3
4. $45 \equiv x \pmod 7$ where $0 \le x < 7$; $x =$
   (a) 2 (b) 3 (c) 4 (d) 5
5. The time 100 hours after 6 a.m. is:
   (a) 6 a.m. (b) 10 a.m. (c) 4 p.m. (d) 2 p.m.
6. The units digit of 2^{2026} is:
   (a) 2 (b) 6 (c) 8 (d) 4
7. The number of equivalence classes modulo 5 is:
   (a) 5 (b) 4 (c) 6 (d) infinite
8. Rice at ₹ 40 and ₹ 60 is mixed to get ₹ 45. The ratio is:
   (a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 1 : 1
9. 40 L of milk, 4 L replaced by water twice; milk left:
   (a) 32 L (b) 30 L (c) 36 L (d) 32.4 L
10. Downstream 15 km/h and upstream 9 km/h; the boat's speed is:
    (a) 3 (b) 12 (c) 6 (d) 24 km/h
11. In Q10, the stream's speed is:
    (a) 6 (b) 12 (c) 3 (d) 4 km/h
12. A fills a tank in 4 h, B in 12 h; together:
    (a) 3 h (b) 2 h (c) 8 h (d) 16 h
13. A pipe fills in 5 h; with a leak in 6 h. The leak empties the tank in:
    (a) 11 h (b) 30 h (c) 1 h (d) 15 h

14. A beats B by 20 m or 4 s; B's speed is:
    (a) 4 m/s (b) 80 m/s (c) 5 m/s (d) 24 m/s

15. A runs twice as fast as B; A gives B 50 m start. The race length for a dead heat is:
    (a) 50 m (b) 200 m (c) 150 m (d) 100 m

16. A invests ₹ 2,000 for 12 months, B ₹ 3,000 for 4 months. Profit ratio:
    (a) 2 : 1 (b) 2 : 3 (c) 1 : 2 (d) 3 : 2

17. $\varphi(10)$ is:
    (a) 4 (b) 5 (c) 3 (d) 6

18. $\mu(30)$ is:
    (a) 1 (b) 3 (c) 0 (d) -1

19. Waiting time equals:
    (a) CT − AT (b) TAT − BT (c) BT − AT (d) CT + BT

20. $17 +_{30} 13$ equals:
    (a) 30 (b) 1 (c) 0 (d) 4

1 (c) · 2 (a) · 3 (d) · 4 (b) · 5 (b) · 6 (d) · 7 (a) · 8 (c) · 9 (d) · 10 (b) · 11 (c) · 12 (a) · 13 (b) · 14 (c) · 15 (d) · 16 (a) · 17 (a) · 18 (d) · 19 (b) · 20 (c)

Assertion–Reason (6)

(a) Both true, R explains A. (b) Both true, R does not explain A. (c) A true, R false. (d) A false, R true.

1. A: $-41 \bmod 9 = 4$. R: The remainder in modulo arithmetic is always non-negative and less than the modulus.
2. A: The units digit of 7^{100} is 7. R: Units digits of powers of 7 repeat in a cycle of 4.
3. A: Rice at ₹ 30 and ₹ 50 mixed 1 : 1 costs ₹ 40/kg. R: The mean price is the weighted average of the prices.
4. A: Downstream speed is greater than the boat's speed in still water. R: The stream adds its speed downstream.
5. A: Two pipes taking 3 h and 6 h fill a tank together in 4.5 h. R: Rates of pipes add.
6. A: In partnership, a partner who invests for a longer time gets a larger share for the same capital. R: Profit is shared in the ratio of capital × time.

Key: 1 (a) · 2 (d) · 3 (a) · 4 (a) · 5 (d) · 6 (a)

Answers to the practice set

A. 1. 2 2. 1 3. 9 4. 9 (cycle 3, 9, 7, 1; $50 \bmod 4 = 2$) 5. 3 6. 0 7. time 8. burst

B. 1. T 2. F 3. T 4. F 5. T 6. F (B starts 20 m ahead) 7. T 8. T

C. 1. $3 \times 4 = 12 \equiv 2$ 2. $50 \equiv 2 \pmod{24}$: 12 noon (two days later) 3. $5 \equiv -1$, $(-1)^{40} = 1$ 4. $(180 - 160) : (160 - 120) = 1 : 2$ 5. 10 km/h 6. $\frac{1}{5}$ 7. 5 m/s 8. $x(1 - \frac{y}{x})^n$ 9. $36 = 2^2 3^2$: $3 \times 3 = 9$ 10. completion time − arrival time

D. 1. $2 \equiv -1$, $(-1)^{100} = 1$ 2. cycle 8, 4, 2, 6; $123 \bmod 4 = 3$: 2 3. $\frac{400+260}{3} =$ ₹ 220 4. down 8, up 6: boat 7, stream 1 km/h 5. $\frac{1}{10} + \frac{1}{15} = \frac{1}{6}$: 6 h 6. B runs 90 m in 10 s while A runs 100 m: A's time 10 s 7. $24000 : 36000 = 2 : 3$: ₹ 2,000 and ₹ 3,000 8. 13, 18, 23, 28

E. 1. 17^{43}: 7-cycle, $43 \bmod 4 = 3$: 3; 23^{22}: 3-cycle, $22 \bmod 4 = 2$: 9; $3 \times 9 = 27$: 7 2. milk 40, water 20; add 20 L 3. down 10, up 6: man 8, stream 2 km/h 4. $\frac{1}{12} + \frac{1}{15} - \frac{1}{20} = \frac{5+4-3}{60} = \frac{1}{10}$: 10 h 5. B 960 when A finishes; C runs

$960 \times \frac{950}{1000} = 912$: 88 m 6. A $20000 \times 6 + 40000 \times 6 = 360000$, B 360000, C 480000: 3 : 3 : 4: ₹ 8,100, 8,100, 10,800

F. 1. 8–9 a.m. A + B: $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$; then all three $\frac{7}{24} + \frac{1}{12} = \frac{9}{24} = \frac{3}{8}$ per hour; remaining $\frac{17}{24} \div \frac{3}{8} = \frac{17}{9}$ h $= 1$ h 53 min: full at about 10:53 a.m. 2. $80(0.9)^3 = 58.32$ L milk, water 21.68 L: ratio $58.32 : 21.68 = 729 : 271$

G. 1. if the remainder were n or more, one more n could be subtracted 2. no: a weighted average always lies between the extremes 3. the slower upstream leg takes extra time that the faster leg does not fully recover: $\frac{2dx}{x^2 - y^2} > \frac{2d}{x}$ 4. the rent is for the house per month, so each month's rent is shared by those present that month, not by "months stayed" alone

Case. (i) $\frac{1}{6} + \frac{1}{9} = \frac{5}{18}$ (ii) $\frac{5}{18} - \frac{1}{12} = \frac{10-3}{36} = \frac{7}{36}$ per hour (iii) remaining $\frac{13}{18} \div \frac{7}{36} = \frac{26}{7}$ h $= 3$ h 43 min: about 10:43 a.m. (iv) $\frac{1}{6} - \frac{1}{12} = \frac{1}{12}$: 12 hours

Last-night summary

- $a \bmod n$ = non-negative remainder; congruence $a \equiv b$ iff $n \mid a - b$; reduce first, then add/multiply; use cycles for powers and units digits.
- Alligation: cheaper : dearer = $(d - m) : (m - c)$; replacement: $x(1 - y/x)^n$.
- Boats: $x \pm y$; pipes: add rates per hour, outlets negative; stage-wise for pipes opened at different times.
- Races: same time ⇒ distance ∝ speed; "by x m or t s" gives the loser's speed.
- Partnership: capital × time, salary first; scheduling: TAT = CT − AT, WT = TAT − BT.