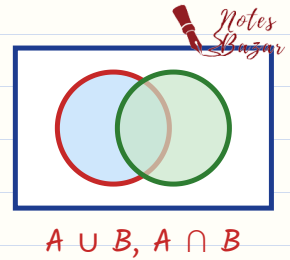


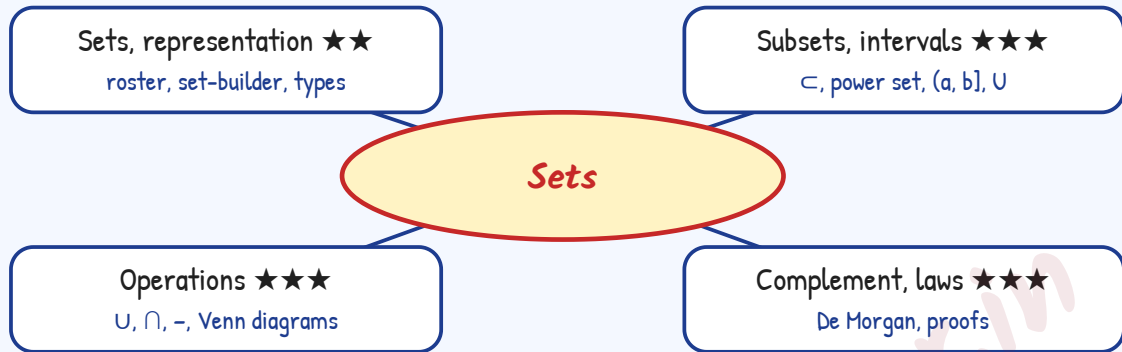
Class 11 · Maths · Chapter 1

# Chapter 1 Sets



NCERT Mathematics, Reprint 2026-27

## Chapter map (★ = very often asked in school exams)



## What is inside

1. Concept notes with all NCERT examples (p. 2-3)
2. NCERT exercises (1.1-1.5 and Miscellaneous), fully solved (p. 4-7)
3. **Important exam questions, fully solved** (p. 8-9)
4. Quick-reference sheet (p. 9); practice A-G, 20 MCQs, 6 A-R, answers, summary (p. 10-13)

## Colour code used

red = key term   green = definition

purple = sub-heading   black = explanation

★ = high-frequency exam topic

! = mark the examiner gives for that point

★ **Exam tip:** Roster and set-builder forms, the difference between  $\in$  and  $\subset$  (with sets as elements), the power set and number of subsets, intervals, union, intersection, difference, complement, De Morgan's laws, and short proofs of set identities are asked again and again.

# 1.2 Sets and their representations; 1.3–1.5 empty, finite, equal sets; 1.6 subsets; 1.7 universal set ★★

## 1.2 Sets

A set is a well-defined collection of objects: we can decide for certain whether a given object belongs to it.

"The five most renowned mathematicians" is not well defined (opinion); "the odd natural numbers less than 10" is. The objects are **elements**:  $a \in A$  ( $a$  belongs to  $A$ ),  $b \notin A$ . Standard sets: **N** natural numbers, **Z** integers, **Q** rationals, **R** reals, **Z<sup>+</sup>**, **Q<sup>+</sup>**, **R<sup>+</sup>** the positive ones.

**Roster (tabular) form**: list the elements in braces, separated by commas, each once, in any order:  $\{1, 2, 3, 6\}$ ; the letters of SCHOOL:  $\{S, C, H, O, L\}$ . **Set-builder form**: state the common property:  $\{x : x \text{ is a natural number and a divisor of } 6\}$ .

**Example 1** Solution set of  $x^2 + x - 2 = 0$ :  $(x - 1)(x + 2) = 0$ , so  $\{1, -2\}$ .

**Example 2**  $\{x : x \text{ a positive integer, } x^2 < 40\} = \{1, 2, 3, 4, 5, 6\}$ .

**Example 3**  $\{1, 4, 9, 16, 25, \dots\} = \{x : x = n^2, n \in \mathbb{N}\}$ .

**Example 4**  $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\} = \{x : x = \frac{n}{n+1}, n \in \mathbb{N}, 1 \leq n \leq 6\}$ .

**Example 5**  $\{P, R, I, N, C, A, L\} \leftrightarrow$  letters of PRINCIPAL;  $\{0\} \leftrightarrow \{x : x + 1 = 1\}$ ;  $\{1, 2, 3, 6, 9, 18\} \leftrightarrow$  divisors of 18;  $\{3, -3\} \leftrightarrow \{x : x^2 - 9 = 0\}$ .

## 1.3–1.5 Empty, finite, infinite, equal sets

The **empty (null, void) set**  $\varnothing = \{\}$  has no element:  $\{x : 1 < x < 2, x \in \mathbb{N}\}$ ,  $\{x : x^2 = 2, x \text{ rational}\}$ . A set is **finite** if it is empty or has a definite number of elements ( $n(S)$  is that number), otherwise **infinite** ( $\mathbb{N}$ , points on a line). Not every infinite set can be written in roster form ( $\mathbb{R}$  cannot).

**Equal sets**:  $A = B$  if they have exactly the same elements (repetition and order do not matter:  $\{1, 2, 2, 3\} = \{3, 1, 2\}$ ).

**Example 6** Finite:  $\{x : x \in \mathbb{N}, (x - 1)(x - 2) = 0\} = \{1, 2\}$ ;  $\{x : x \in \mathbb{N}, x^2 = 4\} = \{2\}$ ;  $\{x : x \in \mathbb{N}, 2x - 1 = 0\} = \varnothing$ . Infinite:  $\{x : x \in \mathbb{N}, x \text{ prime}\}$ ;  $\{x : x \in \mathbb{N}, x \text{ odd}\}$ .

**Example 7**  $A = \{0\}$ ,  $B = \{x : x > 15 \text{ and } x < 5\} = \varnothing$ ,  $C = \{x : x - 5 = 0\} = \{5\}$ ,  $D = \{x : x^2 = 25\} = \{-5, 5\}$ ,  $E = \{x : x \text{ a positive integral root of } x^2 - 2x - 15 = 0\} = \{5\}$ : **C = E** only.

**Example 8**  $X =$  letters of ALLOY =  $\{A, L, O, Y\} =$  letters of LOYAL: **equal**.  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{1, 2\}$ : not equal.

## 1.6 Subsets

$A \subset B$  if every element of  $A$  is an element of  $B$ :  $a \in A \Rightarrow a \in B$ .

Every set is a subset of itself;  $\varnothing$  is a subset of every set.  $A \subset B$  and  $B \subset A \Leftrightarrow A = B$ . If  $A \subset B$  and  $A \neq B$ ,  $A$  is a **proper subset** of  $B$ ,  $B$  a **superset** of  $A$ . A set with one element is a **singleton**.  $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ ;  $\mathbb{T}$  (irrationals)  $\subset \mathbb{R}$ ,  $\mathbb{N} \not\subset \mathbb{T}$ .

**Example 9**  $\varnothing \subset$  every set;  $A = \{1, 3, 5\}$ ,  $B = \{1, 2, 3, 4, 5\}$ :  $A \subset B$  ( $A \neq B$ , proper). **Example 10**  $\{a, e, i, o, u\} \not\subset \{a, b, c, d\}$ . **Example 11** If  $A \in B$  and  $B \subset C$ , is  $A \subset C$ ? **No**:  $A = \{1\}$ ,  $B = \{\{1\}, 2\}$ ,  $C = \{\{1\}, 2, 3\}$ :  $A \in B \subset C$  so  $A \in C$ , but  $1 \notin C$ , so  $A \not\subset C$ .

**Power set**  $P(A)$ : the set of all subsets of  $A$ . If  $n(A) = m$ , then  $n(P(A)) = 2^m$  ( $P(\{1, 2\}) = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$ ).

## Intervals as subsets of R

For  $a < b$ : **open**  $(a, b) = \{x : a < x < b\}$ ; **closed**  $[a, b] = \{x : a \leq x \leq b\}$ ; **half-open**  $[a, b) = \{x : a \leq x < b\}$ ,  $(a, b] = \{x : a < x \leq b\}$ . Also  $(0, \infty) = \mathbb{R}^+$ ,  $(-\infty, 0)$  negative reals,  $(-\infty, \infty) = \mathbb{R}$ . The length of each is  $b - a$ .

## 1.7 Universal set U

The basic set of a context, of which all sets considered are subsets: for integers,  $U$  may be  $\mathbb{Q}$  or  $\mathbb{R}$ ; in population studies, all people.

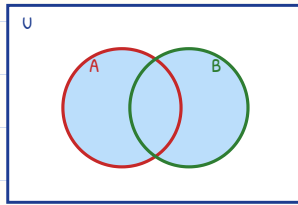
**X Don't do this**:  $\in$  links an **element** to a set;  $\subset$  links a **set** to a set. If  $A = \{1, \{2, 3\}\}$ , then  $\{2, 3\} \in A$  and  $\{\{2, 3\}\} \subset A$ , but  $\{2, 3\} \not\subset A$ .

**X Don't do this**:  $\varnothing$ ,  $\{0\}$  and  $\{\varnothing\}$  are all different:  $\varnothing$  has no element;  $\{0\}$  and  $\{\varnothing\}$  each have one.

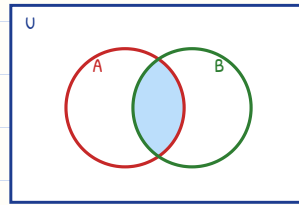
# 1.8 Venn diagrams; 1.9 operations on sets; 1.10 complement; laws

★★★

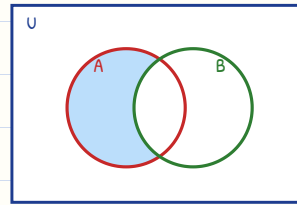
shaded region = the set named below each diagram



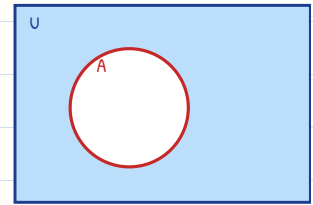
$A \cup B$



$A \cap B$



$A - B$



$A'$  (complement)

$$n(A \cup B) = n(A) + n(B) - n(A \cap B); (A \cup B)' = A' \cap B'; (A \cap B)' = A' \cup B' \text{ (De Morgan)}$$

Venn diagrams of the four basic operations on sets

## 1.9.1 Union

$A \cup B = \{x : x \in A \text{ or } x \in B\}$ : all elements that are in A, in B or in both.

Properties: commutative  $A \cup B = B \cup A$ ; associative  $(A \cup B) \cup C = A \cup (B \cup C)$ ;  $A \cup \phi = A$  (identity);  $A \cup A = A$  (idempotent);  $U \cup A = U$ .

**Example 12**  $\{2, 4, 6, 8\} \cup \{6, 8, 10, 12\} = \{2, 4, 6, 8, 10, 12\}$ .

**Example 13**  $A = \{a, e, i, o, u\}$ ,  $B = \{a, i, u\}$ :  $A \cup B = A$  (since  $B \subset A$ ). **Example 14**  $X = \{\text{Ram, Geeta, Akbar}\}$  (school hockey team),  $Y = \{\text{Geeta, David, Ashok}\}$  (football team):  $X \cup Y = \{\text{Ram, Geeta, Akbar, David, Ashok}\}$ .

## 1.9.2 Intersection

$A \cap B = \{x : x \in A \text{ and } x \in B\}$ : the common elements.

$A \cap B = \phi$ : A and B are **disjoint**. Properties: commutative, associative;  $\phi \cap A = \phi$ ;  $U \cap A = A$ ;  $A \cap A = A$ ; **distributive**:  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .

**Example 15**  $\{2, 4, 6, 8\} \cap \{6, 8, 10, 12\} = \{6, 8\}$ . **Example 16**  $X \cap Y$  (Example 14) = **Geeta**. **Example 17**  $A = \{1, \dots, 10\}$ ,  $B = \{2, 3, 5, 7\}$ :  $A \cap B = \{2, 3, 5, 7\} = B$  ( $B \subset A$ ).

## 1.9.3 Difference

$A - B = \{x : x \in A \text{ and } x \notin B\}$ .

$A - B \neq B - A$  in general;  $A - B$ ,  $A \cap B$  and  $B - A$  are mutually disjoint, and their union is  $A \cup B$ .

**Example 18**  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{2, 4, 6, 8\}$ :  $A - B = \{1, 3, 5\}$ ,  $B - A = \{8\}$ . **Example 19**  $V = \{a, e, i, o, u\}$ ,  $B = \{a, i, k, u\}$ :  $V - B = \{e, o\}$ ,  $B - V = \{k\}$ .

## 1.10 Complement

$A' = \{x : x \in U \text{ and } x \notin A\} = U - A$ .

**Example 20**  $U = \{1, 2, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$ :  $A' = \{2, 4, 6, 8, 10\}$ . **Example 21**  $U$  = all students of a co-educational Class XI,  $A$  = all girls:  $A' =$  **all boys**. **Example 22**  $U = \{1, \dots, 6\}$ ,  $A = \{2, 3\}$ ,  $B = \{3, 4, 5\}$ :  $A \cup B = \{2, 3, 4, 5\}$ ,  $(A \cup B)' = \{1, 6\}$ ;  $A' = \{1, 4, 5, 6\}$ ,  $B' = \{1, 2, 6\}$ ,  $A' \cap B' = \{1, 6\} = (A \cup B)'$ . Likewise  $(A \cap B)' = \{1, 2, 4, 5, 6\} = A' \cup B'$ .

## Laws of complements

| law               | statement                                               |
|-------------------|---------------------------------------------------------|
| complement laws   | $A \cup A' = U$ ; $A \cap A' = \phi$                    |
| De Morgan         | $(A \cup B)' = A' \cap B'$ ; $(A \cap B)' = A' \cup B'$ |
| double complement | $(A')' = A$                                             |
| $\phi$ and $U$    | $\phi' = U$ ; $U' = \phi$                               |

## Miscellaneous examples

**Example 23** Letters of CATARACT:  $X = \{C, A, T, R\}$ ; Letters of TRACT:  $Y = \{T, R, A, C\}$ : every element of X is in Y and vice versa, so  **$X = Y$** .

**Example 24** Subsets of  $\{-1, 0, 1\}$ :  $\phi$ ,  $\{-1\}$ ,  $\{0\}$ ,  $\{1\}$ ,  $\{-1, 0\}$ ,  $\{-1, 1\}$ ,  $\{0, 1\}$ ,  $\{-1, 0, 1\}$  ( $2^3 = 8$ ).

**Example 25**  $A \cup B = A \cap B \Rightarrow A = B$ : if  $a \in A$  then  $a \in A \cup B = A \cap B$ , so  $a \in B$ :  $A \subset B$ ; similarly  $B \subset A$ . Hence  **$A = B$** .

**X Don't do this:** To prove two sets equal, show each is a subset of the other (take an arbitrary element). Venn diagrams illustrate but do not prove.

## NCERT exercises, fully solved

### Exercise 1.1

1. Which of the following are sets? Justify. (i) Months of a year beginning with J (ii) Ten most talented writers of India (iii) A team of eleven best cricket batsmen of the world (iv) All boys in your class (v) All natural numbers less than 100 (vi) Novels written by Munshi Prem Chand (vii) All even integers (viii) The questions in this chapter (ix) The most dangerous animals of the world.

**Ans.** A collection is a set only if membership can be decided definitely. (i) {January, June, July}: **set**. (ii) "Most talented" depends on opinion: **not a set**. (iii) "Best" is a matter of opinion: **not a set**. (iv) **set**. (v) {1, 2, ..., 99}: **set**. (vi) **set** (the novels are known). (vii) **set**. (viii) **set**. (ix) "Most dangerous" is not well defined: **not a set**. So (i), (iv), (v), (vi), (vii), (viii) are sets.

2.  $A = \{1, 2, 3, 4, 5, 6\}$ . Insert  $\in$  or  $\notin$ : (i)  $5 \dots A$  (ii)  $8 \dots A$  (iii)  $0 \dots A$  (iv)  $4 \dots A$  (v)  $2 \dots A$  (vi)  $10 \dots A$ .

**Ans.** (i)  $5 \in A$  (ii)  $8 \notin A$  (iii)  $0 \notin A$  (iv)  $4 \in A$  (v)  $2 \in A$  (vi)  $10 \notin A$ .

3. Write in roster form: (i)  $A = \{x : x \text{ an integer, } -3 \leq x < 7\}$  (ii)  $B = \{x : x \text{ a natural number less than 6}\}$  (iii)  $C = \{x : x \text{ a two-digit natural number whose digits add to 8}\}$  (iv)  $D = \{x : x \text{ a prime divisor of 60}\}$  (v)  $E = \text{letters of TRIGONOMETRY}$  (vi)  $F = \text{letters of BETTER}$ .

**Ans.** (i)  $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$  (7 excluded). (ii)  $\{1, 2, 3, 4, 5\}$ . (iii) Tens digit 1 to 8 with units 8 – tens:  $\{17, 26, 35, 44, 53, 62, 71, 80\}$ . (iv)  $60 = 2^2 \times 3 \times 5$ :  $\{2, 3, 5\}$ . (v)  $\{T, R, I, G, O, N, M, E, Y\}$  (each letter once). (vi)  $\{B, E, T, R\}$ .

4. Write in set-builder form: (i)  $\{3, 6, 9, 12\}$  (ii)  $\{2, 4, 8, 16, 32\}$  (iii)  $\{5, 25, 125, 625\}$  (iv)  $\{2, 4, 6, \dots\}$  (v)  $\{1, 4, 9, \dots, 100\}$ .

**Ans.** (i)  $\{x : x = 3n, n \in \mathbb{N}, 1 \leq n \leq 4\}$ . (ii)  $\{x : x = 2^n, n \in \mathbb{N}, 1 \leq n \leq 5\}$ . (iii)  $\{x : x = 5^n, n \in \mathbb{N}, 1 \leq n \leq 4\}$ . (iv)  $\{x : x \text{ is an even natural number}\}$ . (v)  $\{x : x = n^2, n \in \mathbb{N}, 1 \leq n \leq 10\}$ .

5. List the elements: (i)  $A = \{x : x \text{ an odd natural number}\}$  (ii)  $B = \{x : x \text{ an integer, } -\frac{1}{2} < x < \frac{9}{2}\}$  (iii)  $C = \{x : x \text{ an integer, } x^2 \leq 4\}$  (iv)  $D = \text{letters of LOYAL}$  (v)  $E = \text{months not having 31 days}$  (vi)  $F = \text{consonants preceding k}$ .

**Ans.** (i)  $\{1, 3, 5, \dots\}$ . (ii) Integers strictly between  $-0.5$  and  $4.5$ :  $\{0, 1, 2, 3, 4\}$ . (iii)  $x^2 \leq 4 \Rightarrow -2 \leq x \leq 2$ :  $\{-2, -1, 0, 1, 2\}$ . (iv)  $\{L, O, Y, A\}$ . (v)  $\{\text{February, April, June, September, November}\}$ . (vi)  $\{b, c, d, f, g, h, j\}$ .

6. Match: (i)  $\{1, 2, 3, 6\}$  (ii)  $\{2, 3\}$  (iii)  $\{M, A, T, H, E, I, C, S\}$  (iv)  $\{1, 3, 5, 7, 9\}$  with (a)  $\{x : x \text{ a prime divisor of 6}\}$  (b)  $\{x : x \text{ an odd natural number less than 10}\}$  (c)  $\{x : x \text{ a natural number and divisor of 6}\}$  (d)  $\{x : x \text{ a letter of MATHEMATICS}\}$ .

**Ans.** (i)  $\leftrightarrow$  (c); (ii)  $\leftrightarrow$  (a); (iii)  $\leftrightarrow$  (d); (iv)  $\leftrightarrow$  (b).

### Exercise 1.2

1. Which are null sets? (i) odd natural numbers divisible by 2 (ii) even primes (iii)  $\{x : x \in \mathbb{N}, x < 5 \text{ and } x > 7\}$  (iv)  $\{y : y \text{ a point common to any two parallel lines}\}$ .

**Ans.** (i) No odd number is divisible by 2: **null**. (ii)  $\{2\}$ : **not null**. (iii) No number is both  $< 5$  and  $> 7$ : **null**. (iv) Parallel lines never meet: **null**. Answer: (i), (iii), (iv).

2. Finite or infinite? (i) months of a year (ii)  $\{1, 2, 3, \dots\}$  (iii)  $\{1, 2, \dots, 100\}$  (iv) positive integers greater than 100 (v) primes less than 99.

**Ans.** (i) 12 elements: **finite**. (ii) **infinite**. (iii) 100 elements: **finite**. (iv) **infinite**. (v) **finite** (25 primes).

3. Finite or infinite? (i) lines parallel to the x-axis (ii) letters of the English alphabet (iii) multiples of 5 (iv) animals living on the earth (v) circles through the origin.

**Ans.** (i) **infinite** ( $y = c$  for every real  $c$ ). (ii) **finite** (26). (iii) **infinite**. (iv) **finite** (very large but definite). (v) **infinite** (any centre gives one).

4. Is  $A = B$ ? (i)  $\{a, b, c, d\}, \{d, c, b, a\}$  (ii)  $\{4, 8, 12, 16\}, \{8, 4, 16, 18\}$  (iii)  $\{2, 4, 6, 8, 10\}, \{x : x \text{ a positive even integer, } x \leq 10\}$  (iv)  $\{x : x \text{ a multiple of 10}\}, \{10, 15, 20, 25, 30, \dots\}$ .

**Ans.** (i) Same elements:  $A = B$ . (ii)  $12 \in A$  but  $12 \notin B$ :  $A \neq B$ . (iii)  $B = \{2, 4, 6, 8, 10\}$ :  $A = B$ . (iv)  $15 \in B$  but  $15 \notin A$ :  $A \neq B$ .

5. Are these equal? (i)  $A = \{2, 3\}$ ,  $B = \{x : x \text{ a solution of } x^2 + 5x + 6 = 0\}$  (ii)  $A = \text{letters of FOLLOW}$ ,  $B = \text{letters of WOLF}$ .

**Ans.** (i)  $x^2 + 5x + 6 = (x + 2)(x + 3) = 0$  gives  $B = \{-2, -3\} \neq \{2, 3\}$ : **not equal**. (ii)  $A = \{F, O, L, W\}$ ,  $B = \{W, O, L, F\}$ : **equal**.

6. Select equal sets:  $A = \{2, 4, 8, 12\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{4, 8, 12, 14\}$ ,  $D = \{3, 1, 4, 2\}$ ,  $E = \{-1, 1\}$ ,  $F = \{0, a\}$ ,  $G = \{1, -1\}$ ,  $H = \{0, 1\}$ .

**Ans.**  $B = D$  (both  $\{1, 2, 3, 4\}$ ) and  $E = G$  (both  $\{-1, 1\}$ ); no other pair has the same elements.

### Exercise 1.3

1. Fill  $\subset$  or  $\not\subset$ : (i)  $\{2, 3, 4\} \dots \{1, 2, 3, 4, 5\}$  (ii)  $\{a, b, c\} \dots \{b, c, d\}$  (iii)  $\{x : x \text{ a student of Class XI of your school}\} \dots \{x : x \text{ a student of your school}\}$  (iv)  $\{x : x \text{ a circle in the plane}\} \dots \{x : x \text{ a circle in the same plane with radius 1 unit}\}$  (v)  $\{x : x \text{ a triangle in a plane}\} \dots \{x : x \text{ a rectangle in the plane}\}$  (vi)  $\{x : x \text{ an equilateral triangle in a plane}\} \dots \{x : x \text{ a triangle in the same plane}\}$  (vii)  $\{x : x \text{ an even natural number}\} \dots \{x : x \text{ an integer}\}$ .

**Ans.** (i)  $\subset$ . (ii)  $a$  is not in the second:  $\not\subset$ . (iii)  $\subset$ . (iv) A circle of radius 2 is not in the second:  $\not\subset$ . (v) A triangle is not a rectangle:  $\not\subset$ . (vi)  $\subset$ . (vii)  $\subset$ .

2. True or false? (i)  $\{a, b\} \subset \{b, c, a\}$  (ii)  $\{a, e\} \subset \{x : x \text{ a vowel of the English alphabet}\}$  (iii)  $\{1, 2, 3\} \subset \{1, 3, 5\}$  (iv)  $\{a\} \subset \{a, b, c\}$  (v)  $\{a\} \in \{a, b, c\}$  (vi)  $\{x : x \text{ an even natural number less than } 6\} \subset \{x : x \text{ a natural number which divides } 36\}$ .

**Ans.** (i)  $a, b$  are both in  $\{b, c, a\}$ , so  $\{a, b\} \subset$  it: **False**. (ii)  $a, e$  are vowels: **True**. (iii)  $2 \notin \{1, 3, 5\}$ : **False**. (iv) **True**. (v) The elements are  $a, b, c$ , not  $\{a\}$ : **False**. (vi)  $\{2, 4\}$  and both divide 36: **True**.

3.  $A = \{1, 2, \{3, 4\}, 5\}$ . Which are incorrect and why? (i)  $\{3, 4\} \subset A$  (ii)  $\{3, 4\} \in A$  (iii)  $\{\{3, 4\}\} \subset A$  (iv)  $1 \in A$  (v)  $1 \subset A$  (vi)  $\{1, 2, 5\} \subset A$  (vii)  $\{1, 2, 5\} \in A$  (viii)  $\{1, 2, 3\} \subset A$  (ix)  $\varphi \in A$  (x)  $\varphi \subset A$  (xi)  $\{\varphi\} \subset A$ .

**Ans.** The elements of  $A$  are 1, 2,  $\{3, 4\}$  and 5. **Incorrect:** (i)  $3 \notin A$  (the element is  $\{3, 4\}$ , not 3); (v) 1 is an element, not a set (correct:  $\{1\} \subset A$ ); (vii)  $\{1, 2, 5\}$  is not an element of  $A$  (it is a subset); (viii)  $3 \notin A$ ; (ix)  $\varphi$  is not an element of  $A$ ; (xi)  $\varphi \notin A$ , so  $\{\varphi\} \not\subset A$ . **Correct:** (ii), (iii), (iv), (vi), (x).

4. Write all subsets of (i)  $\{a\}$  (ii)  $\{a, b\}$  (iii)  $\{1, 2, 3\}$  (iv)  $\varphi$ .

**Ans.** (i)  $\varphi, \{a\}$ . (ii)  $\varphi, \{a\}, \{b\}, \{a, b\}$ . (iii)  $\varphi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$  ( $2^3 = 8$ ). (iv)  $\varphi$  only.

5. Write as intervals: (i)  $\{x : x \in \mathbb{R}, -4 < x \leq 6\}$  (ii)  $\{x : x \in \mathbb{R}, -12 < x < -10\}$  (iii)  $\{x : x \in \mathbb{R}, 0 \leq x < 7\}$  (iv)  $\{x : x \in \mathbb{R}, 3 \leq x \leq 4\}$ .

**Ans.** (i)  $(-4, 6]$  (ii)  $(-12, -10)$  (iii)  $[0, 7)$  (iv)  $[3, 4]$ .

6. Write in set-builder form: (i)  $(-3, 0)$  (ii)  $[6, 12]$  (iii)  $(6, 12]$  (iv)  $[-23, 5)$ .

**Ans.** (i)  $\{x : x \in \mathbb{R}, -3 < x < 0\}$  (ii)  $\{x : x \in \mathbb{R}, 6 \leq x \leq 12\}$  (iii)  $\{x : x \in \mathbb{R}, 6 < x \leq 12\}$  (iv)  $\{x : x \in \mathbb{R}, -23 \leq x < 5\}$ .

7. What universal set would you propose for (i) the set of right triangles (ii) the set of isosceles triangles?

**Ans.** For both: **the set of all triangles** (in a plane); the set of all polygons would also do.

8.  $A = \{1, 3, 5\}$ ,  $B = \{2, 4, 6\}$ ,  $C = \{0, 2, 4, 6, 8\}$ . Which can be a universal set for all three? (i)  $\{0, 1, 2, 3, 4, 5, 6\}$  (ii)  $\varphi$  (iii)  $\{0, 1, 2, \dots, 10\}$  (iv)  $\{1, 2, \dots, 8\}$ .

**Ans.**  $U$  must contain  $A \cup B \cup C = \{0, 1, 2, 3, 4, 5, 6, 8\}$ . (i) lacks 8; (ii) empty; (iv) lacks 0. Only (iii).

## Exercise 1.4

1. Find the union: (i)  $X = \{1, 3, 5\}$ ,  $Y = \{1, 2, 3\}$  (ii)  $A = \{a, e, i, o, u\}$ ,  $B = \{a, b, c\}$  (iii)  $A = \{x : x \text{ a natural multiple of } 3\}$ ,  $B = \{x : x \text{ a natural number less than } 6\}$  (iv)  $A = \{x : x \in \mathbb{N}, 1 < x \leq 6\}$ ,  $B = \{x : x \in \mathbb{N}, 6 < x < 10\}$  (v)  $A = \{1, 2, 3\}$ ,  $B = \varphi$ .

**Ans.** (i)  $\{1, 2, 3, 5\}$ . (ii)  $\{a, b, c, e, i, o, u\}$ . (iii)  $A = \{3, 6, 9, \dots\}$ ,  $B = \{1, 2, 3, 4, 5\}$ :  $A \cup B = \{1, 2, 4, 5\}$  together with all multiples of 3 =  $\{x : x = 1, 2, 4, 5 \text{ or a multiple of } 3\}$ . (iv)  $\{2, \dots, 6\} \cup \{7, 8, 9\} = \{x : x \in \mathbb{N}, 1 < x < 10\}$ . (v)  $\{1, 2, 3\}$ .

2.  $A = \{a, b\}$ ,  $B = \{a, b, c\}$ . Is  $A \subset B$ ? What is  $A \cup B$ ?

**Ans.** Yes,  $A \subset B$ ;  $A \cup B = \{a, b, c\} = B$ .

3. If  $A \subset B$ , what is  $A \cup B$ ?

**Ans.** Every element of  $A$  is already in  $B$ , so  $A \cup B = B$ .

4.  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{5, 6, 7, 8\}$ ,  $D = \{7, 8, 9, 10\}$ .

Find (i)  $A \cup B$  (ii)  $A \cup C$  (iii)  $B \cup C$  (iv)  $B \cup D$  (v)  $A \cup B \cup C$  (vi)  $A \cup B \cup D$  (vii)  $B \cup C \cup D$ .

**Ans.** (i)  $\{1, 2, 3, 4, 5, 6\}$  (ii)  $\{1, 2, 3, 4, 5, 6, 7, 8\}$  (iii)  $\{3, 4, 5, 6, 7, 8\}$  (iv)  $\{3, 4, 5, 6, 7, 8, 9, 10\}$  (v)  $\{1, 2, \dots, 8\}$  (vi)  $\{1, 2, \dots, 10\}$  (vii)  $\{3, 4, \dots, 10\}$ .

5. Find the intersection of each pair of question 1.

**Ans.** (i)  $\{1, 3\}$  (ii)  $\{a\}$  (iii)  $\{3, 6, \dots\} \cap \{1, \dots, 5\} = \{3\}$  (iv)  $\varphi$  (v)  $\varphi$ .

6.  $A = \{3, 5, 7, 9, 11\}$ ,  $B = \{7, 9, 11, 13\}$ ,  $C = \{11, 13, 15\}$ ,  $D = \{15, 17\}$ .

Find (i)  $A \cap B$  (ii)  $B \cap C$  (iii)  $A \cap C \cap D$  (iv)  $A \cap C$  (v)  $B \cap D$  (vi)  $A \cap (B \cup C)$  (vii)  $A \cap D$  (viii)  $A \cap (B \cup D)$  (ix)  $(A \cap B) \cap (B \cup C)$  (x)  $(A \cup D) \cap (B \cup C)$ .

**Ans.** (i)  $\{7, 9, 11\}$  (ii)  $\{11, 13\}$  (iii)  $A \cap C = \{11\}$ , then  $\cap D$ :  $\varphi$  (iv)  $\{11\}$  (v)  $\varphi$  (vi)  $B \cup C = \{7, 9, 11, 13, 15\}$ :  $\{7, 9, 11\}$  (vii)  $\varphi$  (viii)  $B \cup D = \{7, 9, 11, 13, 15, 17\}$ :  $\{7, 9, 11\}$  (ix)  $\{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$  (x)  $\{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11, 15\}$ .

7.  $A = \mathbb{N}$ ,  $B =$  even naturals,  $C =$  odd naturals,  $D =$  primes. Find (i)  $A \cap B$  (ii)  $A \cap C$  (iii)  $A \cap D$  (iv)  $B \cap C$  (v)  $B \cap D$  (vi)  $C \cap D$ .

**Ans.** (i)  $B$  (even naturals) (ii)  $C$  (iii)  $D$  (iv) no number is both even and odd:  $\varphi$  (v) the only even prime:  $\{2\}$  (vi)  $\{x : x \text{ an odd prime}\}$ .

8. Which pairs are disjoint? (i)  $\{1, 2, 3, 4\}$  and  $\{x : x \in \mathbb{N}, 4 \leq x \leq 6\}$  (ii)  $\{a, e, i, o, u\}$  and  $\{c, d, e, f\}$  (iii)  $\{x : x \text{ an even integer}\}$  and  $\{x : x \text{ an odd integer}\}$ .

**Ans.** (i) 4 is common: not disjoint. (ii)  $e$  is common: not disjoint. (iii) No integer is both: **disjoint**.

9.  $A = \{3, 6, 9, 12, 15, 18, 21\}$ ,  $B = \{4, 8, 12, 16, 20\}$ ,  $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ ,  $D = \{5, 10, 15, 20\}$ . Find (i)  $A - B$  (ii)  $A - C$  (iii)  $A - D$  (iv)  $B - A$  (v)  $C - A$  (vi)  $D - A$  (vii)  $B - C$  (viii)  $B - D$  (ix)  $C - B$  (x)  $D - B$  (xi)  $C - D$  (xii)  $D - C$ .

**Ans.** (i)  $\{3, 6, 9, 15, 18, 21\}$  (ii)  $\{3, 9, 15, 18, 21\}$  (iii)  $\{3, 6, 9, 12, 18, 21\}$  (iv)  $\{4, 8, 16, 20\}$  (v)  $\{2, 4, 8, 10, 14, 16\}$  (vi)  $\{5, 10, 20\}$  (vii)  $\{20\}$  (viii)  $\{4, 8, 12, 16\}$  (ix)  $\{2, 6, 10, 14\}$  (x)  $\{5, 10, 15\}$  (xi)  $\{2, 4, 6, 8, 12, 14, 16\}$  (xii)  $\{5, 15, 20\}$ .

10.  $X = \{a, b, c, d\}$ ,  $Y = \{f, b, d, g\}$ . Find (i)  $X - Y$  (ii)  $Y - X$  (iii)  $X \cap Y$ .

**Ans.** (i)  $\{a, c\}$  (ii)  $\{f, g\}$  (iii)  $\{b, d\}$ .

11. If  $R$  is the set of real numbers and  $Q$  the set of rational numbers, what is  $R - Q$ ?

**Ans.** The real numbers that are not rational: **the set of irrational numbers**.

12. True or false? Justify. (i)  $\{2, 3, 4, 5\}$  and  $\{3, 6\}$  are disjoint (ii)  $\{a, e, i, o, u\}$  and  $\{a, b, c, d\}$  are disjoint (iii)  $\{2, 6, 10, 14\}$  and  $\{3, 7, 11, 15\}$  are disjoint (iv)  $\{2, 6, 10\}$  and  $\{3, 7, 11\}$  are disjoint.

**Ans.** (i) **False**: 3 is common. (ii) **False**: a is common. (iii) **True**: the intersection is  $\emptyset$ . (iv) **True**: the intersection is  $\emptyset$ .

### Exercise 1.5

1.  $U = \{1, 2, \dots, 9\}$ ,  $A = \{1, 2, 3, 4\}$ ,  $B = \{2, 4, 6, 8\}$ ,  $C = \{3, 4, 5, 6\}$ . Find (i)  $A'$  (ii)  $B'$  (iii)  $(A \cup C)'$  (iv)  $(A \cup B)'$  (v)  $(A')'$  (vi)  $(B - C)'$ .

**Ans.** (i)  $\{5, 6, 7, 8, 9\}$  (ii)  $\{1, 3, 5, 7, 9\}$  (iii)  $A \cup C = \{1, \dots, 6\}$ :  $\{7, 8, 9\}$  (iv)  $A \cup B = \{1, 2, 3, 4, 6, 8\}$ :  $\{5, 7, 9\}$  (v)  $A = \{1, 2, 3, 4\}$  (vi)  $B - C = \{2, 8\}$ :  $\{1, 3, 4, 5, 6, 7, 9\}$ .

2.  $U = \{a, b, c, d, e, f, g, h\}$ . Find the complements of (i)  $\{a, b, c\}$  (ii)  $\{d, e, f, g\}$  (iii)  $\{a, c, e, g\}$  (iv)  $\{f, g, h, a\}$ .

**Ans.** (i)  $\{d, e, f, g, h\}$  (ii)  $\{a, b, c, h\}$  (iii)  $\{b, d, f, h\}$  (iv)  $\{b, c, d, e\}$ .

3. With  $U = \mathbb{N}$ , write the complements of (i) even naturals (ii) odd naturals (iii) positive multiples of 3 (iv) primes (v) naturals divisible by 3 and 5 (vi) perfect squares (vii) perfect cubes (viii)  $\{x : x + 5 = 8\}$  (ix)  $\{x : 2x + 5 = 9\}$  (x)  $\{x : x \geq 7\}$  (xi)  $\{x : x \in \mathbb{N}, 2x + 1 > 10\}$ .

**Ans.** (i) odd naturals (ii) even naturals (iii)  $\{x : x \in \mathbb{N}, x \text{ not a multiple of } 3\}$  (iv) 1 and the composite numbers (v)  $\{x : x \in \mathbb{N}, x \text{ not divisible by } 15\}$  (i.e. not divisible by 3 or not by 5) (vi) naturals that are not perfect squares (vii) not perfect cubes (viii)  $\{3\}' = \mathbb{N} - \{3\} = \{x : x \in \mathbb{N}, x \neq 3\}$  (ix)  $\{2\}' = \mathbb{N} - \{2\} = \{1, 2, 3, 4, 5, 6\}$  (x)  $2x + 5 = 9 \Rightarrow x = 2$ , so the complement is  $\{1, 2, 3, 4\} = \{x : x \in \mathbb{N}, x \leq 9/2\}$ .

4.  $U = \{1, \dots, 9\}$ ,  $A = \{2, 4, 6, 8\}$ ,  $B = \{2, 3, 5, 7\}$ . Verify (i)  $(A \cup B)' = A' \cap B'$  (ii)  $(A \cap B)' = A' \cup B'$ .

**Ans.**  $A' = \{1, 3, 5, 7, 9\}$ ,  $B' = \{1, 4, 6, 8, 9\}$ . (i)  $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$ ,  $(A \cup B)' = \{1, 9\}$ ;  $A' \cap B' = \{1, 9\}$ : equal. (ii)  $A \cap B = \{2\}$ ,  $(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ;  $A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$ : equal.

5. Draw Venn diagrams for (i)  $(A \cup B)'$  (ii)  $A' \cap B'$  (iii)  $(A \cap B)'$  (iv)  $A' \cup B'$ .

**Ans.** Draw  $U$  as a rectangle with two overlapping circles  $A$  and  $B$ . (i) and (ii): shade everything outside both circles (the two diagrams are the same: De Morgan). (iii) and (iv): shade everything except the lens-shaped overlap  $A \cap B$  (again identical).

6.  $U =$  all triangles in a plane. If  $A$  is the set of triangles with at least one angle different from  $60^\circ$ , what is  $A'$ ?

**Ans.**  $A' =$  triangles with no angle different from  $60^\circ$ , i.e. all three angles  $60^\circ$ : the set of equilateral triangles.

7. Fill in: (i)  $A \cup A' = \dots$  (ii)  $\emptyset' \cap A = \dots$  (iii)  $A \cap A' = \dots$  (iv)  $U' \cap A = \dots$

**Ans.** (i)  $U$  (ii)  $\emptyset' = U$ , so  $U \cap A = A$  (iii)  $\emptyset$  (iv)  $U' = \emptyset$ , so  $\emptyset$ .

### Miscellaneous Exercise on Chapter 1

1. Decide which of  $A = \{x : x \in \mathbb{R}, x^2 - 8x + 12 = 0\}$ ,  $B = \{2, 4, 6\}$ ,  $C = \{2, 4, 6, 8, \dots\}$ ,  $D = \{6\}$  are subsets of one another.

**Ans.**  $x^2 - 8x + 12 = (x - 2)(x - 6) = 0$ :  $A = \{2, 6\}$ . Hence  $D \subset A \subset B \subset C$  (so also  $D \subset B$ ,  $D \subset C$ ,  $A \subset C$ ).

2. True or false? Prove if true, give an example if false. (i)  $x \in A, A \in B \Rightarrow x \in B$  (ii)  $A \subset B, B \in C \Rightarrow A \in C$  (iii)  $A \subset B, B \subset C \Rightarrow A \subset C$  (iv)  $A \not\subset B, B \not\subset C \Rightarrow A \not\subset C$  (v)  $x \in A, A \not\subset B \Rightarrow x \in B$  (vi)  $A \subset B, x \notin B \Rightarrow x \notin A$ .

**Ans.** (i) **False**:  $A = \{1\}$ ,  $B = \{\{1\}, 2\}$ :  $1 \in A$ ,  $A \in B$ , but  $1 \notin B$ . (ii) **False**:  $A = \{1\}$ ,  $B = \{1, 2\}$ ,  $C = \{\{1, 2\}, 3\}$ :  $A \subset B$ ,  $B \in C$ , but  $A \notin C$ . (iii) **True**:  $x \in A \Rightarrow x \in B$  ( $A \subset B$ )  $\Rightarrow x \in C$  ( $B \subset C$ ); so  $A \subset C$ . (iv) **False**:  $A = \{1, 2\}$ ,  $B = \{2, 3\}$ ,  $C = \{1, 2, 4\}$ :  $A \not\subset B$ ,  $B \not\subset C$ , yet  $A \subset C$ . (v) **False**:  $A = \{1, 2\}$ ,  $B = \{2, 3\}$ :  $1 \in A$ ,  $A \not\subset B$ , but  $1 \notin B$ . (vi) **True**: if  $x \in A$  then  $x \in B$  ( $A \subset B$ ), contradicting  $x \notin B$ ; so  $x \notin A$ .

3. If  $A \cup B = A \cup C$  and  $A \cap B = A \cap C$ , show that  $B = C$ .

**Ans.**  $B = B \cap (A \cup B) = B \cap (A \cup C) = (B \cap A) \cup (B \cap C)$  (distributive)  $= (A \cap C) \cup (B \cap C)$  (since  $A \cap B = A \cap C$ )  $= (A \cup B) \cap C = (A \cup C) \cap C = C$ . (The first step uses  $B \subset A \cup B$ .)

4. Show that the following four conditions are equivalent: (i)  $A \subset B$  (ii)  $A - B = \emptyset$  (iii)  $A \cup B = B$  (iv)  $A \cap B = A$ .

**Ans.** (i)  $\Rightarrow$  (ii): every  $x \in A$  is in  $B$ , so no element is in  $A$  but not in  $B$ :  $A - B = \emptyset$ . (ii)  $\Rightarrow$  (iii):  $A - B = \emptyset$  means every  $x \in A$  lies in  $B$ , so  $A \cup B \subset B$ ; and  $B \subset A \cup B$  always:  $A \cup B = B$ . (iii)  $\Rightarrow$  (iv):  $A \subset A \cup B = B$ , so every  $x \in A$  is in  $A \cap B$ :  $A \subset A \cap B \subset A$ , hence  $A \cap B = A$ . (iv)  $\Rightarrow$  (i):  $A = A \cap B \subset B$ . So all four are equivalent.

5. Show that if  $A \subset B$ , then  $C - B \subset C - A$ .

**Ans.** Let  $x \in C - B$ :  $x \in C$  and  $x \notin B$ . Since  $A \subset B$ ,  $x \notin B \Rightarrow x \notin A$ . So  $x \in C$  and  $x \notin A$ :  $x \in C - A$ . Hence  $C - B \subset C - A$ .

6. Show that for any sets  $A$  and  $B$ ,  $A = (A \cap B) \cup (A - B)$  and  $A \cup (B - A) = A \cup B$ .

**Ans.**  $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B') = A \cap (B \cup B') = A \cap U = A$ .  $A \cup (B - A) = A \cup (B \cap A') = (A \cup B) \cap (A \cup A') = (A \cup B) \cap U = A \cup B$ .

7. Using properties of sets, show (i)  $A \cup (A \cap B) = A$  (ii)  $A \cap (A \cup B) = A$ .

**Ans.** (i)  $A \cup (A \cap B) = (A \cap U) \cup (A \cap B) = A \cap (U \cup B) = A \cap U = A$ . (ii)  $A \cap (A \cup B) = (A \cap A) \cup (A \cap B) = A \cup (A \cap B) = A$  by (i).

8. Show that  $A \cap B = A \cap C$  need not imply  $B = C$ .

**Ans.** Take  $A = \{1, 2\}$ ,  $B = \{1, 3\}$ ,  $C = \{1, 4\}$ :  $A \cap B = \{1\} = A \cap C$ , but  $B \neq C$ .

9. If  $A \cap X = B \cap X = \varnothing$  and  $A \cup X = B \cup X$  for some set  $X$ , show that  $A = B$ .

**Ans.**  $A = A \cap (A \cup X) = A \cap (B \cup X) = (A \cap B) \cup (A \cap X) = (A \cap B) \cup \varnothing = A \cap B$ . Similarly  $B = B \cap (B \cup X) = B \cap (A \cup X) = (B \cap A) \cup (B \cap X) = A \cap B$ . Hence  $A = B$ .

10. Find sets  $A$ ,  $B$  and  $C$  such that  $A \cap B$ ,  $B \cap C$  and  $A \cap C$  are non-empty sets and  $A \cap B \cap C = \varnothing$ .

**Ans.**  $A = \{1, 2\}$ ,  $B = \{2, 3\}$ ,  $C = \{1, 3\}$ :  $A \cap B = \{2\}$ ,  $B \cap C = \{3\}$ ,  $A \cap C = \{1\}$  (all non-empty), and  $A \cap B \cap C = \varnothing$ .

## Important exam questions, fully solved

Questions of the kind set in school and annual exams, with complete answers and mark steps.

### A. MCQs and Assertion-Reason (1 mark each)

**Exam-style** If  $A = \{1, 2, 3\}$ , the number of elements of  $P(A)$  is (A) 3 (B) 6 (C) 8 (D) 9 [1]

**Ans.**  $2^3 = 8$ . (C).

**Exam-style** Which is the empty set? (A)  $\{0\}$  (B)  $\{\varnothing\}$  (C)  $\{x : x \in \mathbb{N}, x^2 = 9\}$  (D)  $\{x : x \in \mathbb{N}, x + 3 = 3\}$  [1]

**Ans.**  $x = 0$  is not natural. (D).

**Exam-style**  $\{x : x \in \mathbb{R}, -2 \leq x < 5\}$  as an interval is (A)  $(-2, 5)$  (B)  $[-2, 5)$  (C)  $(-2, 5]$  (D)  $[-2, 5]$  [1]

**Ans.**  $-2$  included,  $5$  excluded. (B).

**Exam-style** If  $A \subset B$ , then  $A \cap B'$  equals (A)  $A$  (B)  $B$  (C)  $\varnothing$  (D)  $U$  [1]

**Ans.** No element of  $A$  lies outside  $B$ . (C).

**Exam-style** If  $n(A) = 10$ ,  $n(B) = 15$ ,  $n(A \cap B) = 4$ , then  $n(A \cup B)$  is (A) 25 (B) 21 (C) 29 (D) 19 [1]

**Ans.**  $10 + 15 - 4 = 21$ . (B).

**Exam-style** Assertion (A):  $\{a\} \subset \{a, b, c\}$ . Reason (R):  $a \in \{a, b, c\}$ . [1]

**Ans.** Both true;  $\{a\} \subset$  the set because its only element  $a$  belongs to it. (A).

**Exam-style** Assertion (A): For any sets,  $A - B = A \cap B'$ . Reason (R):  $(A \cup B)' = A' \cup B'$ . [1]

**Ans.** A true; R false (it is  $A' \cap B'$ ). (C).

### B. Short answer (2 marks)

**Exam-style** Write  $\{x : x = (2n - 1)/n, n \in \mathbb{N}, n \leq 4\}$  in roster form. [2]

**Ans.**  $n = 1, 2, 3, 4$  give  $1, 3/2, 5/3, 7/4$  [1]:  $\{1, 3/2, 5/3, 7/4\}$  [1].

**Exam-style** Write  $P(A)$  for  $A = \{1, \{2\}\}$ . [2]

**Ans.**  $A$  has two elements,  $1$  and  $\{2\}$  [1/2]:  $P(A) = \{\varnothing, \{1\}, \{\{2\}\}, \{1, \{2\}\}\}$  [1/2].

**Exam-style** Show that  $A - B$ ,  $A \cap B$  and  $B - A$  are pairwise disjoint. [2]

**Ans.** An element of  $A - B$  is not in  $B$ , so it is in neither  $A \cap B$  nor  $B - A$  [1]; an element of  $B - A$  is not in  $A$ , so it is not in  $A \cap B$  [1].

**Exam-style** If  $U = \{1, 2, \dots, 10\}$ ,  $A = \{x : x \text{ prime}\}$ , find  $A'$  and  $A - A'$ . [2]

**Ans.**  $A = \{2, 3, 5, 7\}$ ;  $A' = \{1, 4, 6, 8, 9, 10\}$  [1];  $A - A' = A = \{2, 3, 5, 7\}$  [1].

**Exam-style** Prove that  $(A \cap B)' = A' \cup B'$ . [2]

**Ans.**  $x \in (A \cap B)' \Leftrightarrow x \notin A \cap B \Leftrightarrow x \notin A \text{ or } x \notin B$  [1]  
 $\Leftrightarrow x \in A' \text{ or } x \in B' \Leftrightarrow x \in A' \cup B'$  [1].

### C. Short answer (3 marks)

**Exam-style** In a class of 50 students, 30 like Maths, 25 like Science and 10 like both. How many like (i) at least one (ii) only Maths (iii) neither? [3]

**Ans.** (i)  $n(M \cup S) = 30 + 25 - 10 = 45$  [1]. (ii) only Maths  $= 30 - 10 = 20$  [1]. (iii) neither  $= 50 - 45 = 5$  [1].

**Exam-style** Show that  $A \cap (B - C) = (A \cap B) - (A \cap C)$ . [3]

**Ans.**  $A \cap (B - C) = A \cap B \cap C'$  [1].  $(A \cap B) - (A \cap C) = (A \cap B) \cap (A \cap C)' = (A \cap B) \cap (A' \cup C') = (A \cap B \cap A') \cup (A \cap B \cap C') = \varnothing \cup (A \cap B \cap C') = A \cap B \cap C'$ . Equal [1].

**Exam-style** List all subsets of  $\{a, b, c\}$  and find how many are proper subsets. [3]

**Ans.**  $\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$  [2]; proper subsets (all except the set itself): 7 [1].

### D. Long answer (5 marks)

**Exam-style**  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ ,  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{5, 6, 7, 8\}$ . (a) Find  $A \cup (B \cap C)$  and  $(A \cup B) \cap (A \cup C)$ , and comment. (b) Verify De Morgan's laws for  $A$  and  $B$ . [5]

**Ans.** (a)  $B \cap C = \{5, 6\}$ ;  $A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}$  [1].  $A \cup B = \{1, \dots, 6\}$ ,  $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$ ; their intersection  $= \{1, \dots, 6\}$  [1]: equal (distributive law) [1/2]. (b)  $A' = \{5, 6, 7, 8\}$ ,  $B' = \{1, 2, 7, 8\}$ .  $(A \cup B)' = \{7, 8\} = A' \cap B'$  [1].  $(A \cap B)' = \{3, 4\}' = \{1, 2, 5, 6, 7, 8\} = A' \cup B'$  [1/2].

### E. Case study (4 marks)

**Case (exam-style):** In a survey of 400 people, 210 read newspaper H, 180 read newspaper E and 100 read both.

(a) How many read at least one? [1] **Ans.**  $210 + 180 - 100 = 290$ .

(b) How many read only H? [1] **Ans.**  $210 - 100 = 110$ .

(c) How many read neither, and how many read exactly one? [2] **Ans.** Neither:  $400 - 290 = 110$ . Exactly one:  $(210 - 100) + (180 - 100) = 110 + 80 = 190$ .

### Quick-reference sheet

| Item                 | Key fact                                                                                           |
|----------------------|----------------------------------------------------------------------------------------------------|
| set                  | well-defined collection; roster or set-builder form                                                |
| empty set            | $\varnothing = \{ \}$ ; subset of every set                                                        |
| subsets              | $A \subset B: a \in A \Rightarrow a \in B$ ; $A = B \Leftrightarrow A \subset B$ and $B \subset A$ |
| power set            | $n(A) = m \Rightarrow n(P(A)) = 2^m$ ; proper subsets $2^m - 1$                                    |
| intervals            | $(a, b)$ , $[a, b]$ , $[a, b)$ , $(a, b]$ ; length $b - a$                                         |
| union / intersection | or / and; commutative, associative, distributive                                                   |
| difference           | $A - B = A \cap B'$                                                                                |
| complement           | $A' = U - A$ ; $A \cup A' = U$ , $A \cap A' = \varnothing$                                         |
| De Morgan            | $(A \cup B)' = A' \cap B'$ ; $(A \cap B)' = A' \cup B'$                                            |
| counting             | $n(A \cup B) = n(A) + n(B) - n(A \cap B)$                                                          |

**X Don't do this:**  $A \in B$  and  $A \subset B$  mean different things: check whether A is an element of B or a part of B.

**X Don't do this:** In roster form never repeat an element.

**Case study (practice): a sports club.** A club has 60 members: 35 play cricket, 28 play football and 12 play both.

(i) How many play at least one game? (ii) How many play only football? (iii) How many play neither? (iv) Show the data on a Venn diagram and verify  $n(C \cup F) = n(C - F) + n(C \cap F) + n(F - C)$ .

## Practice set: test yourself *Exam-style*

Try every question before looking at the answers on page 13. 64 or more out of 80 means you are ready for the exam.

### A. Fill in the blanks (1 mark each)

1. A set with no element is the \_\_\_\_\_ set.
2. A set with exactly one element is a \_\_\_\_\_ set.
3. If  $n(A) = 4$ ,  $n(P(A)) =$  \_\_\_\_\_.
4.  $\{x : 3 < x \leq 7\}$  as an interval is \_\_\_\_\_.
5.  $A \cup A' =$  \_\_\_\_\_.
6. If  $A \subset B$ ,  $A \cap B =$  \_\_\_\_\_.
7.  $A - B = A \cap$  \_\_\_\_\_.
8.  $(A')' =$  \_\_\_\_\_.

### B. True or false? (1 mark each)

1.  $\{1, 2, 2, 3\}$  has four elements.
2.  $\varnothing \subset A$  for every set  $A$ .
3.  $\{\varnothing\}$  is the empty set.
4. Every set is a subset of itself.
5.  $N \subset Z \subset Q \subset R$ .
6.  $A - B = B - A$  always.
7. The set of lines parallel to the x-axis is finite.
8.  $(A \cup B)' = A' \cap B'$ .

### C. Very short answer (1 mark each)

1. Roster form of letters of BETTER.
2. Set-builder form of  $\{2, 4, 8, 16, 32\}$ .
3. Roster form of  $\{x : x \in Z, x^2 \leq 4\}$ .
4. Is  $\{x : x^2 + 5x + 6 = 0\}$  equal to  $\{2, 3\}$ ?
5. Number of subsets of  $\{a, b, c, d\}$ .
6.  $[6, 12]$  in set-builder form.
7.  $R - Q$  is the set of \_\_\_\_\_.
8.  $B \cap D$  for even naturals  $B$  and primes  $D$ .
9. Complement of  $N$ 's even numbers in  $N$ .
10.  $A'$  if  $A =$  triangles with an angle  $\neq 60^\circ$ .

### D. Short answer I (2 marks each)

1. Write all subsets of  $\{1, 2, 3\}$ .
2. Which of  $\{3, 4\} \subset A$ ,  $\{3, 4\} \in A$  are correct for  $A = \{1, 2, \{3, 4\}, 5\}$ ?
3. Find  $A \cup B$  and  $A \cap B$  for  $A = \{2, 4, 6, 8\}$ ,  $B = \{6, 8, 10, 12\}$ .
4. Find  $A - B$  and  $B - A$  for  $A = \{1, \dots, 6\}$ ,  $B = \{2, 4, 6, 8\}$ .
5. Show that  $A \cup B = A \cap B \Rightarrow A = B$ .
6. Give sets with pairwise non-empty intersections but empty triple intersection.
7. Show that  $A \cap B = A \cap C$  need not give  $B = C$ .
8. Verify  $(A \cap B)' = A' \cup B'$  for  $U = \{1, \dots, 6\}$ ,  $A = \{2, 3\}$ ,  $B = \{3, 4, 5\}$ .

**My score** (fill in after checking the answers)

| Section  | A / 8 | B / 8 | C / 10 | D / 16 | E / 18 | F / 20 | G (bonus) | Total / 80 |
|----------|-------|-------|--------|--------|--------|--------|-----------|------------|
| My marks |       |       |        |        |        |        |           |            |

Rough work

### E. Short answer II (3 marks each)

1. Prove that  $A \subset B \Rightarrow C - B \subset C - A$ .
2. Prove  $A = (A \cap B) \cup (A - B)$ .
3. Prove  $A \cup (A \cap B) = A$  using laws of sets.
4. In a group of 70, 37 like coffee, 52 like tea; each likes at least one. How many like both?
5. Find  $(A \cup D) \cap (B \cup C)$  for  $A = \{3, 5, 7, 9, 11\}$ ,  $B = \{7, 9, 11, 13\}$ ,  $C = \{11, 13, 15\}$ ,  $D = \{15, 17\}$ .
6. Show the equivalence of  $A \subset B$  and  $A \cup B = B$ .

### F. Long answer (5 marks each)

1. State and prove De Morgan's laws, and verify them for  $U = \{1, \dots, 9\}$ ,  $A = \{2, 4, 6, 8\}$ ,  $B = \{2, 3, 5, 7\}$ .
2. Show that the four conditions  $A \subset B$ ,  $A - B = \varnothing$ ,  $A \cup B = B$ ,  $A \cap B = A$  are equivalent.

### G. HOTS (higher-order thinking)

1. If  $A \cup B = A \cup C$  and  $A \cap B = A \cap C$ , prove  $B = C$ .
2. How many elements does  $P(P(\varphi))$  have?
3. If  $n(U) = 50$ ,  $n(A) = 20$ ,  $n(B') = 30$ ,  $n(A \cap B) = 8$ , find  $n(A' \cap B')$ .
4. Is it possible for  $A \subset B$  and  $A \in B$  at the same time? Give an example.

## 20 MCQs

1. Which is a set?  
(a) ten talented writers (b) best batsmen (c) months beginning with J (d) dangerous animals
  2. Roster form of  $\{x : x \in \mathbb{Z}, -3 \leq x < 7\}$  has  
(a) 10 elements (b) 9 elements (c) 11 elements (d) 7 elements
  3.  $\{17, 26, 35, \dots, 80\}$  has digit sum  
(a) 7 (b) 10 (c) 9 (d) 8
  4. Letters of TRIGONOMETRY (number):  
(a) 12 (b) 9 (c) 8 (d) 10
  5.  $\{x : x \in \mathbb{Z}, -\frac{1}{2} < x < 9/2\} =$   
(a)  $\{1, 2, 3, 4\}$  (b)  $\{0, 1, 2, 3, 4\}$  (c)  $\{0, 1, 2, 3\}$  (d)  $\{-1, \dots, 4\}$
  6. Null set:  
(a) even primes (b)  $\{\emptyset\}$  (c)  $\{0\}$  (d) odd numbers divisible by 2
  7. Infinite set:  
(a) circles through the origin (b) English letters (c) animals on earth (d) months
  8. Equal sets from Ex. 1.2 Q6:  
(a) A and C (b) F and H (c) B and D (d) A and B
  9. Incorrect for  $A = \{1, 2, \{3, 4\}, 5\}$ :  
(a)  $\{3, 4\} \in A$  (b)  $1 \in A$  (c)  $\emptyset \subset A$  (d)  $\{3, 4\} \subset A$
  10. Subsets of  $\{1, 2, 3\}$ :  
(a) 6 (b) 8 (c) 7 (d) 9
  11.  $\{x \in \mathbb{R} : -12 < x < -10\} =$   
(a)  $[-12, -10]$  (b)  $[-12, -10)$  (c)  $(-12, -10)$  (d)  $(-12, -10]$
  12. Universal set for  $\{1, 3, 5\}, \{2, 4, 6\}, \{0, 2, 4, 6, 8\}$ :  
(a)  $\{0, 1, \dots, 10\}$  (b)  $\{0, \dots, 6\}$  (c)  $\{1, \dots, 8\}$  (d)  $\emptyset$
  13. If  $A \subset B, A \cup B =$   
(a) A (b) B (c)  $\emptyset$  (d) U
  14.  $A \cap (B \cup D)$  for Ex. 1.4 Q6:  
(a)  $\{11\}$  (b)  $\emptyset$  (c)  $\{7, 9, 11\}$  (d)  $\{7, 9, 11, 15\}$
  15.  $B \cap D$  (evens, primes):  
(a)  $\emptyset$  (b) evens (c) primes (d)  $\{2\}$
  16.  $B - C$  in Ex. 1.4 Q9:  
(a)  $\{20\}$  (b)  $\{4, 8\}$  (c)  $\{16, 20\}$  (d)  $\emptyset$
  17.  $(B - C)'$  in Ex. 1.5 Q1:  
(a)  $\{1, 3, 4, 5, 6, 7, 9\}$  (b)  $\{2, 8\}$  (c)  $\{1, 3, 5, 7, 9\}$  (d)  $\{7, 8, 9\}$
  18. Complement of  $\{x \in \mathbb{N} : 2x + 1 > 10\}$ :  
(a)  $\{1, \dots, 5\}$  (b)  $\emptyset$  (c)  $\{5, 6, \dots\}$  (d)  $\{1, 2, 3, 4\}$
  19.  $U' \cap A =$   
(a) A (b)  $\emptyset$  (c) U (d)  $A'$
  20.  $A = \{x : x^2 - 8x + 12 = 0\} =$   
(a)  $\{2, 4\}$  (b)  $\{3, 4\}$  (c)  $\{2, 6\}$  (d)  $\{6\}$
- 1 (c) · 2 (a) · 3 (d) · 4 (b) · 5 (b) · 6 (d) · 7 (a) · 8 (c) · 9 (d) · 10 (b) · 11 (c) · 12 (a) · 13 (b) · 14 (c) · 15 (d) · 16 (a) · 17 (a) · 18 (d) · 19 (b) · 20 (c)

## Assertion-Reason (6)

- (a) Both true, R explains A. (b) Both true, R does not explain A. (c) A true, R false. (d) A false, R true.
1. A:  $\emptyset$  is a subset of every set. R:  $\emptyset$  has no element that could fail to be in the other set.
  2. A:  $\{a, b\} \not\subset \{b, c, a\}$ . R: Order of elements does not matter in a set.
  3. A: The set of even primes is the null set. R: 2 is the only even prime.
  4. A:  $A - B, A \cap B$  and  $B - A$  are mutually disjoint. R: Their union is  $A \cup B$ .
  5. A: If  $A = \{1, 2\}$ ,  $P(A)$  has 4 elements. R:  $n(P(A)) = 2^{n(A)}$ .
  6. A:  $(A \cap B)' = A' \cap B'$ . R: Complementation turns  $\cap$  into  $\cup$ .
- Key: 1 (a) · 2 (d) · 3 (d) · 4 (b) · 5 (a) · 6 (d)

## Answers to the practice set

**A.** 1. empty (null) 2. singleton 3. 16 4.  $\{3, 7\}$  5.  $\cup$  6. A 7.  $B'$  8. A

**B.** 1. F (three) 2. T 3. F 4. T 5. T 6. F 7. F 8. T

**C.** 1.  $\{B, E, T, R\}$  2.  $\{x : x = 2^n, n \in \mathbb{N}, n \leq 5\}$  3.  $\{-2, -1, 0, 1, 2\}$  4. No  $\{-2, -3\}$  5. 16 6.  $\{x \in \mathbb{R} : 6 \leq x \leq 12\}$  7. irrational numbers 8.  $\{2\}$  9. odd naturals 10. equilateral triangles

**D.** 1.  $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$  2. only  $\{3, 4\} \in A$  3.  $\{2, 4, 6, 8, 10, 12\}; \{6, 8\}$  4.  $\{1, 3, 5\}; \{8\}$  5. each element of A lies in A  $\cap B \subset B$  and vice versa 6.  $\{1, 2\}, \{2, 3\}, \{1, 3\}$  7.  $A = \{1, 2\}, B = \{1, 3\}, C = \{1, 4\}$  8. both =  $\{1, 2, 4, 5, 6\}$

**E.** 1.  $x \in C - B \Rightarrow x \notin B \Rightarrow x \notin A$  2.  $= A \cap (B \cup B') = A$  3.  $= A \cap (U \cup B) = A$  4.  $37 + 52 - 70 = 19$  5.  $\{7, 9, 11, 15\}$  6.  $A \subset B \Rightarrow A \cup B \subset B \subset A \cup B$ ; conversely  $A \subset A \cup B = B$

**F.** 1. element-wise proofs;  $(A \cup B)' = \{1, 9\}, (A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$  2. chain  $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv) \Rightarrow (i)$  as in Misc. Q4

**G.** 1.  $B = B \cap (A \cup C) = (A \cap C) \cup (B \cap C) = C \cap (A \cup B) = C$  2.  $P(\varnothing) = \{\varnothing\}; P(\{\varnothing\}) = \{\varnothing, \{\varnothing\}\}$ : 2 elements 3.  $n(B) = 20; n(A \cup B) = 32; n(A' \cap B') = 50 - 32 = 18$  4. Yes:  $A = \{1\}, B = \{1, \{1\}\}$

**Case.** (i)  $35 + 28 - 12 = 51$  (ii)  $28 - 12 = 16$  (iii)  $60 - 51 = 9$  (iv)  $23 + 12 + 16 = 51$

## Last-night summary

- A set is a well-defined collection; roster and set-builder forms;  $\in$  and  $\notin$ .
- Empty, finite, infinite and equal sets; subsets, proper subsets, power set ( $2^n$ ).
- Intervals  $(a, b)$ ,  $[a, b]$ ,  $[a, b)$ ,  $(a, b]$ ; universal set  $U$ .
- Union, intersection, difference; disjoint sets; Venn diagrams.
- Complement  $A' = U - A$ ; complement laws, De Morgan's laws,  $(A')' = A$ .