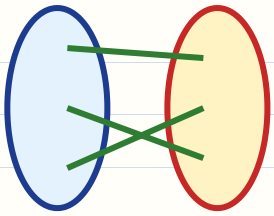
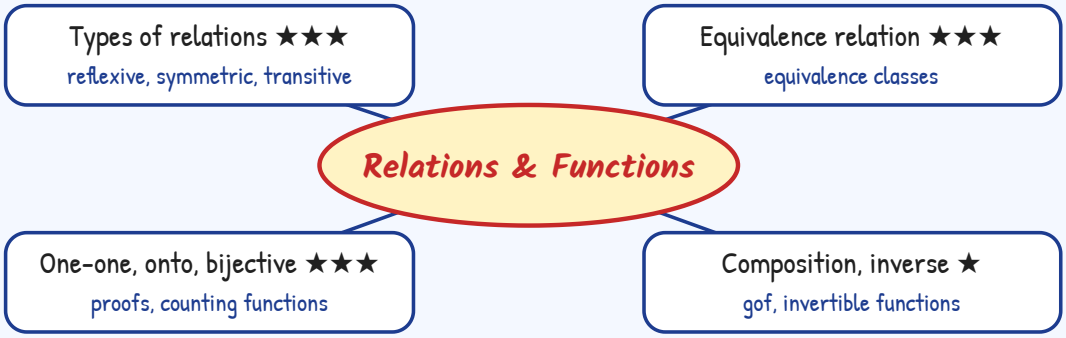
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Chapter 1
Relations and Functions



NCERT Mathematics Part I, Reprint 2026-27 · CBSE board 2027

Chapter map (★ = asked in almost every CBSE paper 2024–2026)



What is inside

1. Relations: types, equivalence relations and classes
2. Functions: one-one, onto, bijective; counting; composition and inverse
3. NCERT Exercises 1.1, 1.2 and Miscellaneous, fully solved
4. CBSE PYQs 2024–2026, fully solved
5. Quick-reference sheet and practice case studies
6. Practice A–G, 20 MCQs, 6 A–R, answers, summary

Colour code used

red = key term green = definition

purple = sub-heading black = working

★ = high-frequency board topic

1 = mark the examiner gives for that step

★ **Exam tip:** This chapter gives **6–8 marks** every year: 1–2 MCQs or A–R, a 2 or 3-mark proof (one-one/onto or equivalence relation), a 5-mark question with choice, and often a full 4-mark case study. Every one of them from the 16 papers is solved.

Based on NCERT Mathematics Part I (2026-27), Chapter 1. Figures drawn by us; all solutions written by us. Board questions are quoted from the CBSE papers named in each tag; every answer checked against the CBSE marking schemes.

1.2 Types of relations ★★★

Definition ✎ A relation R from A to B is any subset of $A \times B$. A relation in (or on) a set A is a subset of $A \times A$. If $(a, b) \in R$ we write $a\,R\,b$.

- **Empty relation**: $R = \phi$ (no element related to any). **Universal relation**: $R = A \times A$. Both are called trivial relations.
- Number of relations from A (m elements) to B (n elements) $= 2^{mn}$; number of relations on A $= 2^{m^2}$ (CBSE 2025 65/2/1, 2026 65/5/1).

Definition ✎ A relation R in A is
reflexive if $(a, a) \in R$ for every $a \in A$;
symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$;
transitive if $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.
R is an **equivalence relation** if it is all three.

How to check, step by step

1. **Reflexive**: test (a, a) for every element. One missing pair is enough to say "not reflexive".
2. **Symmetric**: take a general $(a, b) \in R$ and show $(b, a) \in R$, or give one pair whose reverse is missing.
3. **Transitive**: assume $(a, b), (b, c) \in R$ and prove $(a, c) \in R$; to disprove, give a specific chain $(a, b), (b, c) \in R$ with $(a, c) \notin R$.

✗ Don't do this: A relation with no chain $(a, b), (b, c)$ at all is transitive by default (e.g. $\{(1, 2)\}$). Check carefully before writing "not transitive".

Worked example (NCERT Example 3): lines in a plane, $L_1 R L_2 \iff L_1 \perp L_2$. Not reflexive (no line is perpendicular to itself); symmetric; not transitive ($L_1 \perp L_2$, $L_2 \perp L_3$ make $L_1 \parallel L_3$). Compare: "is parallel to" (counting a line parallel to itself) is an equivalence relation (CBSE 2024 65/5/1).

Equivalence classes

For an equivalence relation R on X, the **equivalence class** of a is $[a] = \{x \in X : x\,R\,a\}$. The classes split X into disjoint pieces whose union is X (a partition): two classes are either equal or disjoint.

Example (NCERT Example 5): on $\mathbb{Z}$, $aRb \iff 2 \mid (a - b)$. Reflexive ($2 \mid 0$), symmetric ($2 \mid (a - b) \Rightarrow 2 \mid (b - a)$), transitive ($a - c = (a - b) + (b - c)$). $[0]$ = even integers, $[1]$ = odd integers.

Example: on $\mathbb{Z}$, $aRb \iff n \mid (a - b)$ (congruence mod n) is an equivalence relation with n classes $[0], [1], \ldots, [n - 1]$ (CBSE 2024 65/2/1, 2026 65/1/1, 2026 65/4/1).

Example: on $A = \{-4, \ldots, 4\}$, $xRy \iff x + y$ even: $[2] = \{-4, -2, 0, 2, 4\}$ (CBSE 2024 65/3/1).

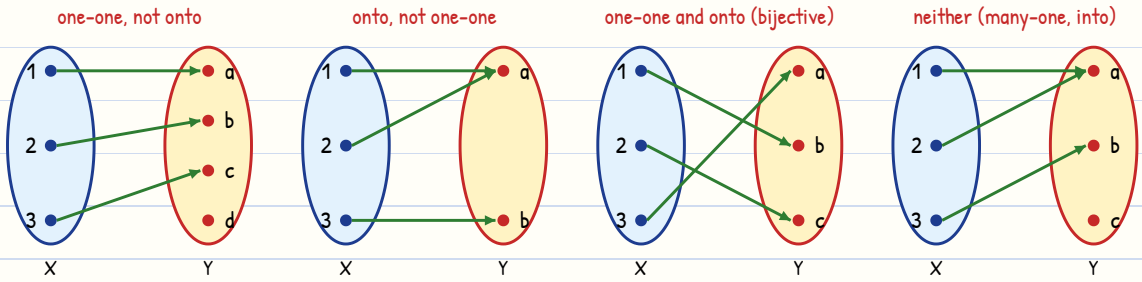
Smallest relations with given properties

To make a given R reflexive add all missing (a, a); to make it symmetric add the reverse of every pair; then add pairs forced by transitivity. On $\{1, 2, 3\}$ with $R = \{(1, 2)\}$: equivalence relations containing $(1, 2)$ are $\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ and $A \times A$: **2** of them (NCERT Misc 7).

Relation on a set of lines/triangles/people	R	S	T
is parallel to	✓	✓	✓
is perpendicular to	✗	✓	✗
is congruent / similar to	✓	✓	✓

$a \le b$ on R	✓	✗	✓
$a < b$ on R	✗	✗	✓
is father of / wife of	✗	✗	✗
is a multiple of (on N)	✓	✗	✓

1.3 Types of functions ★★★ · 1.4 Composition and inverse



One-one: different inputs give different outputs. Onto: every element of the co-domain is hit. Bijective: both (and then f has an inverse)

Definition ✎ **One-one (injective)**: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ (distinct inputs, distinct images). Otherwise **many-one**.
Onto (surjective): every y in the co-domain is $f(x)$ for some x, i.e. range = co-domain. Otherwise **into**.
Bijective: both one-one and onto.

Proving one-one

1. Assume $f(x_1) = f(x_2)$; simplify to $x_1 = x_2$. Example: $f(x) = \frac{x-2}{x-3}$: cross-multiply, $-3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$.
2. Or show $f'(x) > 0$ (or < 0) throughout: a strictly monotonic function is one-one (e.g. $x^3 + 5x + 1$).
3. To disprove, give two different inputs with the same image ($f(-1) = f(1)$ for x^2).

Proving onto

1. Take any y in the co-domain, solve $y = f(x)$ for x.
2. Check that this x lies in the domain (and avoids excluded values).
3. Verify $f(x) = y$. To disprove, find one y with no pre-image (e.g. $y = 0$ for $f(x) = 4x + 3$ on R_+, CBSE 2024 65/1/1).

★ **Exam tip:** For $f : A \to A$ with A **finite**, one-one $\iff$ onto. For infinite sets this fails: $f(x) = 2x$ on N is one-one but not onto.

Counting (sets of sizes $m \to n$)

Number of	Formula
relations A→B	2^{mn}
functions A→B	n^m
one-one functions ($m \le n$)	${}^nP_m = \frac{n!}{(n-m)!}$; 0 if $m > n$
bijections (only if $m = n$)	$n!$
onto functions $\{1, \ldots, n\} \to$ itself	$n!$ (NCERT Misc 4)

So there is no one-one function from a 4-element set to a 3-element set (CBSE 2025 65/2/1), and no onto function from a 3-element set to a 4-element set (CBSE 2026 65/5/1).

Composition and inverse

For $f : A \to B, g : B \to C$: $(g \circ f)(x) = g(f(x))$. In general $g \circ f \neq f \circ g$: with $f = \cos x, g = 3x^2$, $g \circ f = 3\cos^2 x$ but $f \circ g = \cos 3x^2$.

$f : X \to Y$ is **invertible** if there is $g : Y \to X$ with $g \circ f = I_X$ and $f \circ g = I_Y$; then $g = f^{-1}$. **f is invertible $\iff$ f is bijective.** To find f^{-1}: write $y = f(x)$, solve for x.

Example (NCERT Ex 1.1): $f(x) = 4x + 3$, $N \to Y$ (Y = range): $f^{-1}(y) = \frac{y-3}{4}$.

Standard results: x^2 on R neither; x^3 on R bijective; $|x|$, $[x]$, signum on R neither one-one nor onto; $\frac{1}{x}$ on R^* bijective; $\log_a x$, $R^+ \to R$ bijective (CBSE 2025 65/4/1).

NCERT exercises, fully solved

Exercise 1.1

1. Determine whether each relation is reflexive, symmetric and transitive: (i) on $A = \{1, 2, \ldots, 14\}$, $R = \{(x, y) : 3x - y = 0\}$ (ii) on N, $R = \{(x, y) : y = x + 5,\ x < 4\}$ (iii) on $\{1, \ldots, 6\}$, $R = \{(x, y) : y \text{ divisible by } x\}$ (iv) on Z, $R = \{(x, y) : x - y \text{ is an integer}\}$ (v) on human beings: (a) work at the same place (b) live in the same locality (c) x is exactly 7 cm taller than y (d) x is wife of y (e) x is father of y.

> **Ans.** (i) $R = \{(1,3), (2,6), (3,9), (4,12)\}$. $(1,1) \notin R$: not reflexive; $(1,3) \in R$, $(3,1) \notin R$: not symmetric; $(1,3), (3,9) \in R$ but $(1,9) \notin R$: not transitive.
> (ii) $R = \{(1,6), (2,7), (3,8)\}$: not reflexive ($(1,1) \notin R$), not symmetric ($(6,1) \notin R$); transitive (no chain $(a,b), (b,c)$ exists, as no first entry equals a second entry).
> (iii) Reflexive ($x \mid x$); not symmetric ($1 \mid 2$, $2 \nmid 1$); transitive ($x \mid y, y \mid z \Rightarrow x \mid z$).
> (iv) $x - x = 0 \in Z$; $x - y \in Z \Rightarrow y - x \in Z$; $x - y,\ y - z \in Z \Rightarrow x - z \in Z$: equivalence relation.
> (v)(a), (b): reflexive, symmetric, transitive. (c) Not reflexive, not symmetric, not transitive (x 7 cm taller than y, y 7 cm taller than z gives x 14 cm taller). (d) Not reflexive, not symmetric (if x is wife of y, y is husband of x), transitive vacuously (wife of y, and y wife of z cannot both hold). (e) None of the three.

2. Show that $R = \{(a, b) : a \leq b^2\}$ on R is neither reflexive nor symmetric nor transitive.

> **Ans.** $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ since $\frac{1}{2} > \frac{1}{4}$: not reflexive. $(1, 4) \in R$ $(1 \leq 16)$ but $(4, 1) \notin R$ $(4 > 1)$: not symmetric. $(3, 2), (2, 1.5) \in R$ $(3 \leq 4, 2 \leq 2.25)$ but $(3, 1.5) \notin R$ $(3 > 2.25)$: not transitive.

3. Check $R = \{(a, b) : b = a + 1\}$ on $\{1, \ldots, 6\}$.

> **Ans.** $R = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$. Not reflexive ($(1,1) \notin R$); not symmetric ($(2,1) \notin R$); not transitive ($(1,2), (2,3) \in R$, $(1,3) \notin R$).

4. Show that $R = \{(a, b) : a \leq b\}$ on R is reflexive and transitive but not symmetric.

> **Ans.** $a \leq a$: reflexive. $a \leq b, b \leq c \Rightarrow a \leq c$: transitive. $(1,2) \in R$, $(2,1) \notin R$: not symmetric.

5. Check $R = \{(a, b) : a \leq b^3\}$ on R.

> **Ans.** $\left(\frac{1}{2}, \frac{1}{2}\right)$: $\frac{1}{2} \leq \frac{1}{8}$ false: not reflexive. $(1,2) \in R$, $(2,1) \notin R$: not symmetric. $(3, \frac{3}{2}), (\frac{3}{2}, \frac{6}{5}) \in R$ $(3 \leq 3.375, 1.5 \leq 1.728)$ but $3 \leq (1.2)^3 = 1.728$ is false: not transitive.

6. Show that $R = \{(1, 2), (2, 1)\}$ on $\{1, 2, 3\}$ is symmetric but neither reflexive nor transitive.

> **Ans.** $(1,1) \notin R$: not reflexive. Both $(1,2), (2,1)$ present: symmetric. $(1,2), (2,1) \in R$ but $(1,1) \notin R$: not transitive.

7. Books in a library, $xRy \iff$ same number of pages: show R is an equivalence relation.

> **Ans.** A book has as many pages as itself; "same number" is symmetric; if x, y and y, z have the same number of pages, so do x, z. Equivalence relation.

8. On $A = \{1, 2, 3, 4, 5\}$, $R = \{(a, b) : |a - b| \text{ even}\}$: show R is an equivalence relation, elements of $\{1, 3, 5\}$ are related to each other, so are those of $\{2, 4\}$, and none of the first set is related to the second.

> **Ans.** $|a - a| = 0$ even; $|a - b| = |b - a|$; if $|a - b|$ and $|b - c|$ are even, a, b, c have the same parity, so $|a - c|$ is even. Any two odd numbers differ by an even number, as do two even numbers; an odd and an even number differ by an odd number. So $\{1, 3, 5\}$ and $\{2, 4\}$ are the two equivalence classes.

9. On $A = \{0, 1, \ldots, 12\}$ show that (i) $R = \{(a, b) : |a - b| \text{ multiple of } 4\}$ (ii) $R = \{(a, b) : a = b\}$ are equivalence relations; find the elements related to 1.

> **Ans.** (i) $0 = 4 \cdot 0$; $|a - b| = |b - a|$; $a - b = 4k, b - c = 4l \Rightarrow a - c = 4(k + l)$. Related to 1: $\{1, 5, 9\}$. (ii) Equality is reflexive, symmetric, transitive. Related to 1: $\{1\}$.

10. Give examples of relations which are (i) symmetric only (ii) transitive only (iii) reflexive and symmetric, not transitive (iv) reflexive and transitive, not symmetric (v) symmetric and transitive, not reflexive.

> **Ans.** On $\{1, 2, 3\}$: (i) $\{(1, 2), (2, 1)\}$ (ii) $\{(1, 2)\}$ (iii) $\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ (iv) $\{(1, 1), (2, 2), (3, 3), (1, 2)\}$ (v) $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$ ($(3, 3)$ missing). On R, "$a \leq b$" is an example of (iv).

11. Points P, Q in a plane, $PRQ \iff OP = OQ$: equivalence relation; the points related to $P \neq O$ form the circle through P with centre O.

> **Ans.** $OP = OP$; $OP = OQ \Rightarrow OQ = OP$; $OP = OQ, OQ = OS \Rightarrow OP = OS$. The points Q with $OQ = OP$ are exactly the circle with centre O, radius OP.

12. Triangles, $T_1 R T_2 \iff T_1 \sim T_2$: equivalence relation. Which of $T_1(3, 4, 5)$, $T_2(5, 12, 13)$, $T_3(6, 8, 10)$ are related?

> **Ans.** Similarity is reflexive, symmetric and transitive. $\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$, so $T_1 \sim T_3$; T_2 is not similar to either ($\frac{3}{5} \neq \frac{4}{12}$). Only T_1 and T_3 are related.

13. Polygons, $P_1 R P_2 \iff$ same number of sides: equivalence relation; elements related to the triangle with sides 3, 4, 5?

> **Ans.** Equality of the number of sides is reflexive, symmetric, transitive. Related to the triangle: **all triangles.**

14. Lines in the XY-plane, $L_1 R L_2 \iff L_1 \parallel L_2$: equivalence relation; lines related to $y = 2x + 4$?

> **Ans.** Every line is parallel to itself (by convention), parallelism is symmetric and transitive. Related lines: all lines of slope 2, $y = 2x + c, c \in R$.

15. R on $\{1, 2, 3, 4\}$: $\{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. (A) reflexive, symmetric, not transitive (B) reflexive, transitive, not symmetric (C) symmetric, transitive, not reflexive (D) equivalence.

> **Ans.** All (a, a) present: reflexive. $(1, 2) \in R$, $(2, 1) \notin R$: not symmetric. Chains: $(1, 3), (3, 2) \to (1, 2)$ ✓; others give pairs already in R: transitive. **(B).**

16. R on N: $\{(a, b) : a = b - 2,\ b > 6\}$. (A) $(2, 4) \in R$ (B) $(3, 8) \in R$ (C) $(6, 8) \in R$ (D) $(8, 7) \in R$

> **Ans.** Need $b > 6$ and $a = b - 2$: $(6, 8)$ ✓. ($(2, 4)$: $b = 4 \not> 6$; $(3, 8)$: $3 \neq 6$.) **(C).**

Exercise 1.2

1. Show that $f : R^* \to R^*$, $f(x) = \frac{1}{x}$ is one-one and onto. Is it true if the domain is N (co-domain R^*)?

> **Ans.** $\frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$: one-one. For $y \in R^*$, $x = \frac{1}{y} \in R^*$ and $f(x) = y$: onto. With domain N: still one-one, but not onto (e.g. $y = 2$ needs $x = \frac{1}{2} \notin N$).

2. Injectivity and surjectivity of (i) $x^2, N \to N$ (ii) $x^2, Z \to Z$ (iii) $x^2, R \to R$ (iv) $x^3, N \to N$ (v) $x^3, Z \to Z$.

> **Ans.** (i) One-one (positive inputs); not onto (2 is not a square). (ii) Not one-one ($f(-1) = f(1)$); not onto (-1 has no pre-image). (iii) Not one-one; not onto (-2). (iv) One-one ($x_1^3 = x_2^3 \Rightarrow x_1 = x_2$); not onto (2 is not a cube). (v) One-one; not onto (2 has no integer cube root).

3. Prove $f(x) = [x], R \to R$ is neither one-one nor onto.

> **Ans.** $[1.2] = [1.9] = 1$: not one-one. $[x]$ is always an integer, so 0.5 has no pre-image: not onto.

4. Show $f(x) = |x|, R \to R$ is neither one-one nor onto.

> **Ans.** $|-1| = |1|$: not one-one. $|x| \geq 0$, so -1 has no pre-image: not onto.

5. Show the signum function ($1, 0, -1$ for $x > 0, = 0, < 0$) is neither one-one nor onto.

> **Ans.** $f(1) = f(2) = 1$: not one-one. Range $\{-1, 0, 1\} \neq R$ (e.g. 2 has no pre-image): not onto.

6. $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}, f = \{(1, 4), (2, 5), (3, 6)\}$: show f is one-one.

> **Ans.** The images 4, 5, 6 of 1, 2, 3 are all different: one-one. (Not onto, as 7 has no pre-image.)

7. (i) $f(x) = 3 - 4x$ (ii) $f(x) = 1 + x^2, R \to R$: one-one, onto, bijective?

> **Ans.** (i) $3 - 4x_1 = 3 - 4x_2 \Rightarrow x_1 = x_2$; for any y, $x = \frac{3-y}{4}$ gives $f(x) = y$: bijective. (ii) $f(1) = f(-1)$: not one-one; $1 + x^2 \geq 1$, so 0 has no pre-image: not onto.

8. Show $f : A \times B \to B \times A, f(a, b) = (b, a)$ is bijective.

> **Ans.** $(b_1, a_1) = (b_2, a_2) \Rightarrow a_1 = a_2, b_1 = b_2$: one-one. Any $(b, a) \in B \times A$ is $f(a, b)$: onto.

9. $f : N \to N, f(n) = \frac{n+1}{2}$ (n odd), $\frac{n}{2}$ (n even). Is f bijective?

> **Ans.** $f(1) = 1 = f(2)$: not one-one, so not bijective. (It is onto: $m = f(2m)$.)

10. $A = R - \{3\}, B = R - \{1\}, f(x) = \frac{x-2}{x-3}$: one-one and onto?

> **Ans.** One-one: $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6 \Rightarrow x_1 = x_2$. Onto: $y = \frac{x-2}{x-3} \Rightarrow x = \frac{3y-2}{y-1}$, defined for $y \neq 1$, and $x \neq 3$ (it would need $-2 = -3$); $f(x) = y$. So f is one-one and onto (CBSE 2025 65/7/1 and 2026 65/4/1 asked exactly this).

11. $f(x) = x^4, R \to R$: (A) one-one onto (B) many-one onto (C) one-one not onto (D) neither one-one nor onto.

> **Ans.** $f(1) = f(-1)$: many-one; $x^4 \geq 0$: not onto. **(D).**

12. $f(x) = 3x, R \to R$: choose the correct answer from the same options.

> **Ans.** $3x_1 = 3x_2 \Rightarrow x_1 = x_2$; $y = 3 \cdot \frac{y}{3}$: one-one onto. **(A).**

Miscellaneous Exercise on Chapter 1

1. Show $f : R \to \{x : -1 < x < 1\}, f(x) = \frac{x}{1+|x|}$ is one-one and onto.

> **Ans.** For $x \geq 0$, $f(x) = \frac{x}{1+x} \in [0, 1)$; for $x < 0$, $f(x) = \frac{x}{1-x} \in (-1, 0)$. So $f(x_1) = f(x_2)$ forces the same sign; then $\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2} \Rightarrow x_1 = x_2$ (similarly for negatives): one-one. Onto: for $0 \leq y < 1$, $x = \frac{y}{1-y} \geq 0$; for $-1 < y < 0$, $x = \frac{y}{1+y} < 0$; in each case $f(x) = y$.

2. Show $f(x) = x^3, R \to R$ is injective.

> **Ans.** $x_1^3 = x_2^3 \Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$; the second factor is positive unless $x_1 = x_2 = 0$, so $x_1 = x_2$.

3. On $P(X)$, $ARB \iff A \subset B$. Is R an equivalence relation?

> **Ans.** Reflexive ($A \subset A$) and transitive, but not symmetric: $\phi \subset X$ while $X \not\subset \phi$ (X non-empty). Not an equivalence relation.

4. Number of onto functions from $\{1, 2, \ldots, n\}$ to itself.

> **Ans.** On a finite set onto $\iff$ one-one, so these are the bijections (arrangements of n objects): $n!$.

5. $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$, $f(x) = x^2 - x$, $g(x) = 2\left|x - \frac{1}{2}\right| - 1$. Are f and g equal?

> **Ans.** $f(-1) = 2, g(-1) = 2(\frac{3}{2}) - 1 = 2$; $f(0) = 0 = g(0)$; $f(1) = 0 = g(1)$; $f(2) = 2 = g(2)$. Same values at every point of A: $f = g$.

7. Number of equivalence relations on $\{1, 2, 3\}$ containing $(1, 2)$: (A) 1 (B) 2 (C) 3 (D) 4

> **Ans.** $\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ and $A \times A$. **(B).**

6. $A = \{1, 2, 3\}$: number of relations containing $(1, 2)$, $(1, 3)$ which are reflexive and symmetric but not transitive: (A) 1 (B) 2 (C) 3 (D) 4

> **Ans.** Must contain $(1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1)$; this is not transitive ($(2,1), (1,3)$ but $(2,3) \notin$). Adding $(2,3), (3,2)$ gives $A \times A$, transitive. Only one such relation. **(A).**

Quick-reference sheet

Property	Test	To disprove
Reflexive	$(a, a) \in R$ for all a	one missing (a, a)
Symmetric	$(a, b) \Rightarrow (b, a)$	a pair whose reverse is missing
Transitive	$(a, b), (b, c) \Rightarrow (a, c)$	a chain whose end pair is missing
One-one	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$	$f(p) = f(q), p \neq q$
Onto	solve $y = f(x)$, x in domain	a y with no pre-image

✗ **Don't do this:** Writing "R is reflexive" after checking only the pairs that appear. You must check every element of the set.

✗ **Don't do this:** Proving onto without checking that the solved x lies in the domain (e.g. $x = \frac{y}{2}$ is not in N when y is odd).

✗ **Don't do this:** Forgetting the co-domain: $f(x) = x^2$ is onto $[0, \infty)$ but not onto R.

Case study (practice) 1: hostel rooms. A hostel has 4 students $S = \{s_1, s_2, s_3, s_4\}$ and 3 rooms $R = \{r_1, r_2, r_3\}$. Each student is given exactly one room.
(i) How many functions are possible from S to R? (ii) Can such an allotment be one-one? Why? (iii) How many onto allotments are possible if room r_1 must take two students? (iv) The relation "lives in the same room as" on S: is it an equivalence relation?

Case study (practice) 2: the encoder. A code machine sends each integer x to $f(x) = 5x - 7$.
(i) Is $f : Z \to Z$ one-one? (ii) Is it onto? (iii) For $f : R \to R$, find f^{-1}. (iv) Which integers are codes of some integer?

CBSE board questions 2024–2026, fully solved

Every Relations and Functions question from the 16 papers (sets 65/1/1 to 65/7/1, 2024–2026), quoted in full with complete working.

A. MCQs and Assertion–Reason (1 mark each)

CBSE 2024 · 65/1/1 A function $f : R_+ \to R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is (A) one-one but not onto (B) onto but not one-one (C) both one-one and onto (D) neither one-one nor onto [1]

> **Ans.** $4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$: one-one. For $x \geq 0$, $f(x) \geq 3$, so values like $0 \in R$ are never taken: not onto. **(A)**.

CBSE 2024 · 65/2/1 Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is (A) one-one (B) onto (C) bijective (D) neither one-one nor onto [1]

> **Ans.** $f(x) = (3x + 1)^2 - 6$. For $x \geq 0$ it is strictly increasing ($f' = 18x + 6 > 0$): one-one. Its values run from $f(0) = -5$ to ∞, the whole co-domain: onto. **(C)**.

CBSE 2024 · 65/3/1 Let R_+ denote the set of all non-negative real numbers. Then the function $f : R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is (A) one-one but not onto (B) onto but not one-one (C) both one-one and onto (D) neither one-one nor onto [1]

> **Ans.** For $x_1, x_2 \geq 0$, $x_1^2 + 1 = x_2^2 + 1 \Rightarrow x_1 = x_2$: one-one. But $f(x) \geq 1$, so $0, \frac{1}{2} \in R_+$ have no pre-image: not onto. **(A)**.

CBSE 2024 · 65/4/1 Assertion (A): The relation $R = \{(x, y) : (x + y) \text{ is a prime number and } x, y \in N\}$ is not a reflexive relation. Reason (R): The number '$2n$' is composite for all natural numbers n. [1]

> **Ans.** $(2, 2) \notin R$ because 4 is not prime: R is not reflexive, so A is true. R is false: for $n = 1$, $2n = 2$ is prime. **(C)**.

CBSE 2024 · 65/5/1 A function $f : R \to R$ defined as $f(x) = x^2 - 4x + 5$ is (A) injective but not surjective. (B) surjective but not injective. (C) both injective and surjective. (D) neither injective nor surjective. [1]

> **Ans.** $f(0) = f(4) = 5$: not injective. $f(x) = (x - 2)^2 + 1 \geq 1$, so values below 1 are never taken: not surjective. **(D)**.

CBSE 2025 · 65/1/1 Assertion (A): Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5, \forall x \in Z$ is a bijective. Reason (R): A function is a bijective if it is both surjective and injective. [1]

> **Ans.** f is one-one ($3x_1 - 5 = 3x_2 - 5 \Rightarrow x_1 = x_2$) but not onto: $f(x) = 0$ needs $x = \frac{5}{3} \notin Z$. So A is false. R is the definition of a bijection, true. **(D)**.

CBSE 2025 · 65/4/1 For real x, let $f(x) = x^3 + 5x + 1$. Then (A) f is one-one but not onto on R (B) f is onto on R but not one-one (C) f is one-one and onto on R (D) f is neither one-one nor onto on R [1]

> **Ans.** $f'(x) = 3x^2 + 5 > 0$, so f is strictly increasing, hence one-one. It is a cubic, continuous, with $f(x) \to \pm\infty$ as $x \to \pm\infty$, so it takes every real value: onto. **(C)**.

CBSE 2025 · 65/4/1 If $f : N \to W$ is defined as $f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$, then f is (A) injective only (B) surjective only (C) a bijection (D) neither surjective nor injective [1]

> **Ans.** $f(1) = f(3) = 0$, so not injective. Every $w \in W$ is an image: $0 = f(1)$ and $w = f(2w)$ for $w \geq 1$. So f is surjective only. **(B)**.

CBSE 2025 · 65/5/1 Assertion (A): Let $A = \{x \in R : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function. Reason (R): If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$. [1]

> **Ans.** $x^2 \geq 0$, so negative values of the co-domain (like -1) have no pre-image: not onto. R shows exactly this for $y = -1$. Both true, R explains A. **(A)**.

CBSE 2025 · 65/6/1 Assertion (A): Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)x = e^x + \log x$ where domain of $(f + g)$ is R. Reason (R): $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$. [1]

> **Ans.** $\text{Dom}(f) = R, \text{Dom}(g) = (0, \infty)$, so $\text{Dom}(f + g) = (0, \infty)$, not R. A is false; R is true. **(D)**.

CBSE 2026 · 65/2/1 A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (2, 2), (1, 2)\}$ is (A) Reflexive only (B) Reflexive and Transitive (C) Symmetric and Transitive (D) Transitive only [1]

> **Ans.** Not reflexive: $(3, 3) \notin R$. Not symmetric: $(1, 2) \in R$, $(2, 1) \notin R$. Transitive: the only chains are $(1, 1), (1, 2) \to (1, 2)$ and $(1, 2), (2, 2) \to (1, 2)$, both in R. **(D)**.

CBSE 2026 · 65/4/1 Assertion (A): A function $f : N \to N$ given by $f(x) = x^3 + 2, \forall x \in N$ is one-one but not onto. Reason (R): Since $\forall y \in N$ (Codomain), there does not exist $x = (y - 2)^{1/3}$ in N (Domain) such that $f(x) = x^3 + 2 = y$. [1]

> **Ans.** One-one: $x_1^3 = x_2^3 \Rightarrow x_1 = x_2$. Not onto: e.g. $y = 1$ needs $x = (-1)^{1/3} = -1 \notin N$. A true. R gives the reason (the needed $x = (y - 2)^{1/3}$ is not in N for many y, such as $y = 1, 2, 4$). Scheme answer **(A)**. (Read literally "for every y" is too strong, since $y = 3$ gives $x = 1$; the scheme accepts R as the explanation.)

CBSE 2026 · 65/5/1 Assertion (A): A relation R on the set $\{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ is an equivalence relation. Reason (R): A relation that is reflexive, symmetric and transitive is an equivalence relation. [1]

> **Ans.** R contains $(1, 1), (2, 2), (3, 3)$ (reflexive), contains $(2, 1)$ with $(1, 2)$ (symmetric), and every chain stays inside (e.g. $(1, 2), (2, 1) \to (1, 1)$): transitive. A true; R is the definition used to verify it. **(A)**.

B. Very short answer (2 marks)

CBSE 2025 · 65/7/1 Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$. Discuss the bijectivity of the function. [2]

> **Ans.** One-one: $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3) \Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$ ✓. Onto: for $y \in B$, $y = \frac{x-2}{x-3} \Rightarrow x = \frac{3y-2}{y-1}$, which exists as $y \neq 1$ and is never 3 (that would need $-2 = -3$); and $f(x) = y$ ✓. So f is a bijection.

CBSE 2026 · 65/2/1 (a) Check whether $f : R - \{3\} \to R$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not. OR (b) Check whether $f : Z \times Z \to Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not. [2]

Ans. (a) $y = \frac{x-2}{x-3} \Rightarrow x = \frac{3y-2}{y-1}$, which needs $y \neq 1$ (1). So 1 is in the co-domain R but not in the range $R - \{1\}$: f is not onto (1).
OR (b) $f(x_1, y_1) = f(x_2, y_2) \Rightarrow (2y_1, 3x_1) = (2y_2, 3x_2) \Rightarrow y_1 = y_2, x_1 = x_2$ (1). So $(x_1, y_1) = (x_2, y_2)$: injective (1).

C. Short answer (3 marks)

CBSE 2024 · 65/1/1 (a) A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive. OR (b) A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not. [3]

Ans. (a) Reflexive: $|x^2 - x^2| = 0 < 8$ ✓ (1). Symmetric: $|x^2 - y^2| = |y^2 - x^2|$ ✓ (1). Not transitive: $(1, 2) \in R$ ($|1 - 4| = 3$), $(2, 3) \in R$ ($|4 - 9| = 5$), but $(1, 3) \notin R$ ($|1 - 9| = 8 \not< 8$) (1).
OR (b) $a + b = 1, 2a + b = 3 \Rightarrow a = 2, b = -1$: $f(x) = 2x - 1$ (1). One-one: $2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$ (1). Onto: for $y \in R$, $x = \frac{y+1}{2} \in R$ gives $f(x) = y$ (1).

CBSE 2025 · 65/2/1 Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, $m, n \in N$." Find whether R is reflexive, symmetric and transitive or not. [3]

Ans. Reflexive: $m = 1 \cdot m$, so mRm for every m ✓ (1). Symmetric: $4R2$ ($4 = 2 \times 2$) but 2 is not a multiple of 4, so not symmetric ✗ (1). Transitive: $m = kn$ and $n = lp$ ($k, l \in N$) give $m = (kl)p$, so mRp ✓ (1). R is reflexive and transitive but not symmetric.

CBSE 2025 · 65/4/1 (a) If $f : R^+ \to R$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection. (R^+ is a set of all positive real numbers.) OR (b) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6,\ x \in A,\ y \in B\}$. (i) Write all elements of R. (ii) Is R a function? Justify. (iii) Determine domain and range of R. [3]

Ans. (a) One-one: $f(x_1) = f(x_2) \Rightarrow \log_a x_1 = \log_a x_2 \Rightarrow x_1 = x_2$ (1½). Onto: for any $y \in R$, $x = a^y \in R^+$ and $f(x) = \log_a a^y = y$ (1½). Hence f is a bijection.
OR (b)(i) $1 + 5 = 6, 2 + 4 = 6$ ($3 + 3$ fails as $3 \notin B$): $R = \{(1, 5), (2, 4)\}$ (1). (ii) Not a function from A to B: $3 \in A$ has no image (1). (iii) Domain $= \{1, 2\}$, range $= \{4, 5\}$ (1).

CBSE 2025 · 65/6/1 (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not. OR (b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is even} \\ n + 1, & \text{if } n \text{ is odd} \end{cases}$ is a bijection. [3]

Ans. (a) $y = 41 - 2x \geq 1 \Rightarrow x \leq 20$: $R = \{(1, 39), (2, 37), \ldots, (20, 1)\}$. Domain $= \{1, 2, \ldots, 20\}$, range $= \{1, 3, 5, \ldots, 39\}$ (1). Not reflexive: $(1, 1) \notin R$ ($2 + 1 \neq 41$). Not

symmetric: $(1,39) \in R$ but $(39,1) \notin R$. Not transitive: $(11,19),(19,3) \in R$ but $(11,3) \notin R$ $(22+3 \neq 41)$ 1½. So R is not an equivalence relation ½.
OR (b) One-one: if n_1, n_2 are both even or both odd, $f(n_1) = f(n_2)$ gives $n_1 = n_2$. If n_1 is odd and n_2 even, $f(n_1) = n_1 + 1$ is even and $f(n_2) = n_2 - 1$ is odd, so they cannot be equal 1½. Onto: an odd $m \in N$ is $f(m+1)$ ($m+1$ even); an even m is $f(m-1)$ ($m-1 \geq 1$ odd) 1½. Hence f is a bijection.

CBSE 2025 · 65/7/1 (a) Show that the function $f : R \to R$ defined by $f(x) = 4x^3 - 5, \forall x \in R$ is one-one and onto. OR (b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x,y) : xy \text{ is a square of a natural number}, x, y \in N\}$. Determine if the relation R is an equivalence relation. [3]

Ans. (a) One-one: $4x_1^3 - 5 = 4x_2^3 - 5 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$ 1½. Onto: for $y \in R$, $x = \left(\frac{y+5}{4}\right)^{1/3} \in R$ and $f(x) = 4 \cdot \frac{y+5}{4} - 5 = y$ 1½.
OR (b) Reflexive: $x \cdot x = x^2$ ✓ ½. Symmetric: $xy = yx$, so $(x,y) \in R \Rightarrow (y,x) \in R$ ✓ ½. Transitive: $xy = a^2, yz = b^2 \Rightarrow xy^2z = a^2b^2 \Rightarrow xz = \left(\frac{ab}{y}\right)^2$. Since xz is a natural number whose square root $\frac{ab}{y}$ is rational, it is a natural number, so xz is a perfect square ✓ 1½. R is an equivalence relation ½.

CBSE 2026 · 65/4/1 (a) Let $A = R - \{3\}$ and $B = R - \{1\}$. A function $f : A \to B$ is defined by $f(x) = \frac{x-2}{x-3}$. Find whether f is one-one and onto. OR (b) Let n be a fixed positive integer. A relation R is defined in set Z such that $R = \{(x,y) : (x-y) \text{ is divisible by } n, x, y \in Z\}$. Determine if R is an equivalence relation. [3]

Ans. (a) One-one: $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$ 1½. Onto: for $y \neq 1$, $x = \frac{3y-2}{y-1} \in A$ (it is never 3) and $f(x) = y$ 1½. So f is one-one and onto.
OR (b) Reflexive: $x - x = 0 = n \cdot 0$ 1. Symmetric: $x - y = nk \Rightarrow y - x = n(-k)$ 1. Transitive: $x - y = nk, y - z = nl \Rightarrow x - z = n(k+l)$ 1. R is an equivalence relation.

CBSE 2024 · 65/3/1 A relation R on set $A = \{-4,-3,-2,-1,0,1,2,3,4\}$ be defined as $R = \{(x,y) : x + y \text{ is an integer divisible by } 2\}$. Show that R is an equivalence relation. Also, write the equivalence class [2]. [5]

Ans. Reflexive: $x + x = 2x$ is even ✓ 1. Symmetric: $x + y$ even $\Rightarrow y + x$ even ✓ 1. Transitive: $x + y$ and $y + z$ even $\Rightarrow x + z = (x+y) + (y+z) - 2y$ even ✓ 2. So R is an equivalence relation. $[2] = \{y \in A : 2 + y \text{ even}\} = \{-4,-2,0,2,4\}$ 1.

D. Long answer (5 marks)

CBSE 2024 · 65/2/1 (a) Show that a function $f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function. OR (b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as: $(a,b)\,R\,(c,d) \iff a - c = b - d$. Show that R is an equivalence relation. [5]

Ans. (a) $f(2) = \frac{4}{5} = f\left(\frac{1}{2}\right)$ with $2 \neq \frac{1}{2}$: not one-one 1½. $y = \frac{2x}{1+x^2} \Rightarrow yx^2 - 2x + y = 0$; real x needs $4 - 4y^2 \geq 0$, i.e. $-1 \leq y \leq 1$ 2. So e.g. $2 \in R$ has no pre-image: not onto. With $A = [-1,1]$, $f : R \to A$ is onto 1½.

OR (b) Reflexive: $a-a=b-b$, so $(a,b)R(a,b)$ 1. Symmetric: $a-c=b-d \Rightarrow c-a=d-b$, so $(c,d)R(a,b)$ 1½. Transitive: $a-c=b-d$ and $c-e=d-f$; adding, $a-e=b-f$, so $(a,b)R(e,f)$ 2. Hence R is an equivalence relation ½.

CBSE 2024 · 65/4/1 (a) Let $A=R-\{5\}$ and $B=R-\{1\}$. Consider the function $f:A\to B$, defined by $f(x)=\frac{x-3}{x-5}$. Show that f is one-one and onto. OR (b) Check whether the relation S in the set of real numbers R defined by $S=\{(a,b):\text{where } a-b+\sqrt{2} \text{ is an irrational number}\}$ is reflexive, symmetric or transitive. [5]

Ans. (a) One-one: $\frac{x_1-3}{x_1-5}=\frac{x_2-3}{x_2-5}\Rightarrow(x_1-3)(x_2-5)=(x_2-3)(x_1-5)\Rightarrow -5x_1-3x_2=-5x_2-3x_1\Rightarrow x_1=x_2$ 2½. Onto: $y=\frac{x-3}{x-5}\Rightarrow x=\frac{5y-3}{y-1}$, defined for $y\neq 1$ and never equal to 5 (that would need $-3=-5$); $f(x)=y$ 2½.
OR (b) Reflexive: $a-a+\sqrt{2}=\sqrt{2}$ irrational ✓ 1. Not symmetric: $(\sqrt{2},0)\in S\,(2\sqrt{2})$ but $(0,\sqrt{2})\notin S\,(0)$ 2. Not transitive: $(\sqrt{2},1)\in S\,(2\sqrt{2}-1)$, $(1,2\sqrt{2})\in S\,(1-\sqrt{2})$, but $(\sqrt{2},2\sqrt{2})\notin S\,(0)$ 2.

CBSE 2026 · 65/1/1 (a) A relation R is defined on Z, the set of integers, as $R=\{(x,y):|x-y| \text{ is divisible by a prime number } p,\ x,y\in Z\}$; check whether R is an equivalence relation or not. OR (b) A function $f:R-\left\{\frac{3}{5}\right\}\to R-\left\{\frac{3}{5}\right\}$ is defined as $f(x)=\frac{3x+2}{5x-3}$. Show that f is one-one and onto. [5]

Ans. (a) Reflexive: $|x-x|=0=p\cdot 0$ ✓ 1. Symmetric: $|x-y|=|y-x|$ ✓ 1½. Transitive: $x-y=pk, y-z=pl\Rightarrow x-z=p(k+l)$, divisible by p ✓ 2. R is an equivalence relation ½.
OR (b) One-one: $\frac{3x_1+2}{5x_1-3}=\frac{3x_2+2}{5x_2-3}\Rightarrow 15x_1x_2-9x_1+10x_2-6=15x_1x_2-9x_2+10x_1-6\Rightarrow 19x_1=19x_2$ 2½. Onto: $y=\frac{3x+2}{5x-3}\Rightarrow x=\frac{3y+2}{5y-3}$, defined as $y\neq\frac{3}{5}$, and $x\neq\frac{3}{5}$ (that would need $10=-9$); then $f(x)=y$ 2½.

CBSE 2026 · 65/3/1 Show that $f:R\to R$ defined as $f(x)=\frac{x}{\sqrt{1+x^2}}$ is one-one but not onto. [5]

Ans. One-one: if $f(x_1)=f(x_2)$, then x_1, x_2 have the same sign and $\frac{x_1^2}{1+x_1^2}=\frac{x_2^2}{1+x_2^2}\Rightarrow x_1^2=x_2^2$, so $x_1=x_2$ 2½. (Also $f'(x)=\frac{1}{(1+x^2)^{3/2}}>0$.) Not onto: $|f(x)|=\frac{|x|}{\sqrt{1+x^2}}<1$ for all x, so e.g. $y=1$ has no pre-image ($x=\frac{y}{\sqrt{1-y^2}}$ needs $|y|<1$); range $(-1,1)\neq R$ 2½.

E. Case studies (4 marks)

CBSE 2024 · 65/5/1, Q36 (case, 4 marks): Students of a school are taken to a railway museum to learn about railways heritage and its history. An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by $R=\{(l_1,l_2):l_1 \text{ is parallel to } l_2\}$.

(i) Find whether the relation R is symmetric or not. [1] **Ans.** If $l_1\parallel l_2$ then $l_2\parallel l_1$: symmetric.

(ii) Find whether the relation R is transitive or not. [1] **Ans.** If $l_1\parallel l_2$ and $l_2\parallel l_3$ then $l_1\parallel l_3$: transitive.

(iii)(a) If one of the rail lines on the railway track is represented by the equation $y=3x+2$, then find the set of rail lines in R related to it. [2] **Ans.** All lines with the same slope 3: $\{y=3x+c:c\in R\}$.

OR (iii)(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$; check whether the relation S is symmetric and transitive. [2] **Ans.** Symmetric: $l_1 \perp l_2 \Rightarrow l_2 \perp l_1$. Not transitive: $l_1 \perp l_2$ and $l_2 \perp l_3$ make $l_1 \parallel l_3$, not perpendicular.

CBSE 2025 · 65/1/1, Q37 (case, 4 marks): A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$: $R_1 = \{(2, 3), (3, 2)\}$, $R_2 = \{(1, 2), (1, 3), (3, 2)\}$, $R_3 = \{(1, 2), (2, 1), (1, 1)\}$, $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$, $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$.

(i) Identify the relation which is reflexive, transitive but not symmetric. [1] **Ans.** R_4: it contains $(1, 1), (2, 2), (3, 3)$; $(1, 2) \in R_4$ but $(2, 1) \notin R_4$; every chain such as $(1, 2), (2, 2)$ gives $(1, 2) \in R_4$.

(ii) Identify the relation which is reflexive and symmetric but not transitive. [1] **Ans.** R_5: reflexive and symmetric, but $(1, 2), (2, 3) \in R_5$ while $(1, 3) \notin R_5$.

(iii)(a) Identify the relations which are symmetric but neither reflexive nor transitive. [2] **Ans.** R_1 (not reflexive: $(1, 1) \notin R_1$; $(2, 3), (3, 2)$ but $(2, 2) \notin R_1$) and R_3 ($(2, 2) \notin R_3$; $(2, 1), (1, 2)$ but $(2, 2) \notin R_3$).

OR (iii)(b) What pairs should be added to the relation R_2 to make it an equivalence relation? [2] **Ans.** Reflexive: add $(1, 1), (2, 2), (3, 3)$; symmetric: add $(2, 1), (3, 1), (2, 3)$. The result is $A \times A$, which is transitive. Pairs to add: $(1, 1), (2, 2), (3, 3), (2, 1), (3, 1), (2, 3)$.

CBSE 2025 · 65/2/1, Q36 (case, 4 marks): A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y,\ x \in S,\ y \in J\}$.

(i) How many relations can be there from S to J? [1] **Ans.** $n(S \times J) = 4 \times 3 = 12$, so the number of relations $= 2^{12} = 4096$.

(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$. Check if it is bijective. [1] **Ans.** $f(S_2) = f(S_3) = J_2$, so f is not one-one (though it is onto). Not bijective.

(iii)(a) How many one-one functions can be there from set S to set J? [2] **Ans.** A one-one function needs 4 different images, but J has only 3 elements. Number of one-one functions $= 0$.

OR (iii)(b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. [2] **Ans.** Add $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$. Then R_1 is reflexive, and since $(S_1, S_2) \in R_1$ but $(S_2, S_1) \notin R_1$ it is not symmetric.

CBSE 2025 · 65/5/1, Q37 (case, 4 marks): Let A be the set of 30 students of class XII in a school. Let $f : A \to N$, N is a set of natural numbers such that function $f(x) =$ Roll Number of student x.

(i) Is f a bijective function? [1] **Ans.** No.

(ii) Give reasons to support your answer to (i). [1] **Ans.** Different students have different roll numbers,

so f is one-one; but the range is $\{1, 2, \ldots, 30\} \neq N$ (co-domain), so f is not onto, hence not bijective.

(iii)(a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where $R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = 3x\}$. List the elements of R. Is the relation R reflexive, symmetric and transitive? Justify your answer. [2] **Ans.** $R = \{(1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21), (8, 24), (9, 27), (10, 30)\}$. Not reflexive: $(1, 1) \notin R$. Not symmetric: $(1, 3) \in R$ but $(3, 1) \notin R$. Not transitive: $(1, 3), (3, 9) \in R$ but $(1, 9) \notin R$.

OR (iii)(b) Let R be a relation defined by $R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = x^3\}$. List the elements of R. Is R a function? Justify your answer. [2] **Ans.** $R = \{(1, 1), (2, 8), (3, 27)\}$. Not a function on the set of roll numbers: the elements $4, 5, \ldots, 30$ have no image.

CBSE 2026 · 65/5/1, Q36 (case, 4 marks): 7 students out of 80 are selected for a 'Sustainability' project and are categorized into two sets: girl students $A = \{G_1, G_2, G_3, G_4\}$, boy students $B = \{B_1, B_2, B_3\}$.

(i) How many relations are possible from Set A $\rightarrow$ Set B? [1] **Ans.** $n(A \times B) = 12$, so $2^{12} = 4096$ relations.

(ii) Let R be a relation from A $\rightarrow$ B such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function? Justify your answer. [1] **Ans.** No. It is not even a function: G_1 has two images, B_1 and B_2.

(iii)(a) Let the relation R from A $\rightarrow$ A be such that $R = \{(x, y) : x, y \in A,$ x and y are students from the same colony in the city$\}$. Verify if R is an equivalence relation. [2] **Ans.** Reflexive: every student is from the same colony as herself. Symmetric: if x and y are from the same colony, so are y and x. Transitive: if x, y and y, z share a colony, then x and z share it. So R is an equivalence relation.

OR (iii)(b) Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer. [2] **Ans.** No. B has 3 elements and A has 4, so at most 3 elements of A can be images: no function from B to A is onto, hence none is bijective.

What the board asked (16 papers, 2024–2026)

Type	2024	2025	2026	Total
MCQs and Assertion–Reason (1 mark each)	5	5	3	13
Very short answer (2 marks)	0	1	1	2
Short answer (3 marks)	1	4	1	6
Long answer (5 marks)	3	0	2	5
Case studies (4 marks)	1	3	1	5

Practice set: test yourself *Exam-style*

Try every question before looking at the answers at the end of the chapter. 64 or more out of 80 means you are ready for the board.

A. Fill in the blanks (1 mark each)

1. The number of relations on a set with 3 elements is _____.
2. A relation that is reflexive, symmetric and transitive is an _____ relation.
3. $f(x) = x^3$ on R is _____ (one-one/many-one).
4. The number of bijections from $\{1, 2, 3, 4\}$ to itself is _____.
5. $f : X \to Y$ is onto if range $=$ _____.
6. On Z, $aRb \iff 5 \mid (a - b)$ has _____ equivalence classes.
7. $f(x) = |x|$ on R is _____ (onto / not onto).
8. If $f(x) = 2x + 3$, then $f^{-1}(x) =$ _____.

B. True or false? (1 mark each)

1. The empty relation on a non-empty set is reflexive.
2. "Is perpendicular to" on lines is symmetric.
3. Every one-one function from a finite set to itself is onto.
4. $f(x) = x^2$ is one-one on $[0, \infty)$.
5. $\{(1, 2)\}$ on $\{1, 2\}$ is transitive.
6. Number of functions from a 2-element set to a 3-element set is 8.
7. Two equivalence classes are either equal or disjoint.
8. $f(x) = \sin x$, $R \to R$ is onto.

C. Very short answer (1 mark each)

1. Define a symmetric relation.
2. Give an example of a relation that is transitive only.
3. Find $[3]$ for $aRb \iff 4 \mid (a - b)$ on $\{0, \ldots, 10\}$.
4. Is $f(x) = x^2 + 1$, $R \to R$ onto?
5. How many one-one functions from a 2-set to a 4-set?
6. If $f(x) = x^2$, $g(x) = x + 1$, find $g \circ f(2)$.
7. Write the range of the signum function.
8. When is a function invertible?
9. Find the number of relations from $\{a, b\}$ to $\{1, 2, 3\}$.
10. Is "$a < b$" on R reflexive?

D. Short answer I (2 marks each)

1. Show that $f(x) = 7x - 2$, $R \to R$ is bijective.
2. Check whether $R = \{(a, b) : a - b \text{ is odd}\}$ on Z is transitive.
3. Show that $f : N \to N$, $f(x) = x + 3$ is not onto.
4. Find all equivalence classes of $aRb \iff 3 \mid (a - b)$ on $\{1, \ldots, 7\}$.
5. Show that $f(x) = \frac{x}{x+1}$, $x \neq -1$ is one-one.
6. Find $g \circ f$ and $f \circ g$ for $f(x) = |x|$, $g(x) = |5x - 2|$.

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7. Give a relation on $\{1,2,3\}$ that is reflexive and symmetric but not transitive.
8. Is $f: Z \to Z$, $f(x) = 2x$ onto? Justify.

My score (fill in after checking the answers)

Section	A /8	B /8	C /10	D /16	E /18	F /20	G (bonus)	Total /80
My marks								

Rough work

E. Short answer II (3 marks each)

1. Show that $R = \{(a,b) : a+b \text{ even}\}$ on $\mathbb{Z}$ is an equivalence relation; write $[0]$.
2. Show that $f: R - \{2\} \to R - \{1\}$, $f(x) = \frac{x-1}{x-2}$ is a bijection.
3. On $N \times N$, $(a,b)R(c,d) \iff a+d = b+c$: prove R is an equivalence relation.
4. Show that $f(x) = \frac{x}{x^2+1}$, $R \to R$ is neither one-one nor onto.
5. Let $A = \{1,2,3\}$. Write the smallest equivalence relation containing $(1,2)$ and $(2,3)$.
6. Show that $f: N \to N$, $f(n) = n+1$ (n odd), $n-1$ (n even) is bijective.

F. Long answer (5 marks each)

1. (a) On the set of real numbers, $aRb \iff a - b + \sqrt{3}$ is irrational. Check reflexive, symmetric, transitive. (b) Show that $f: R \to R$, $f(x) = \frac{2x-1}{3}$ is invertible and find f^{-1}.
2. (a) Show that $f: [-1,1] \to R$, $f(x) = \frac{x}{x+2}$ is one-one; find the range and the inverse $f: [-1,1] \to \text{Range}$. (b) How many functions, one-one functions and onto functions are there from $\{1,2,3\}$ to $\{a,b\}$?

G. HOTS (higher-order thinking)

1. Can a relation be symmetric and transitive but not reflexive? Give an example.
2. Why is every strictly increasing function one-one?
3. If $f \circ g$ is one-one, must g be one-one?
4. How many reflexive relations are there on a set of 3 elements?

20 MCQs

1. Number of relations on $\{1,2\}$
   (a) 4 (b) 8 (c) 16 (d) 2

2. $\{(1,1),(2,2)\}$ on $\{1,2,3\}$ is
   (a) symmetric and transitive (b) reflexive (c) an equivalence (d) universal

3. $f(x) = 2x$, $N \to N$ is
   (a) onto (b) many-one (c) bijective (d) one-one, not onto

4. $f(x) = x^2$, $R \to R$ is
   (a) one-one (b) neither one-one nor onto (c) onto (d) bijective

5. Bijections on a 3-element set
   (a) 3 (b) 6 (c) 9 (d) 27

6. "Is perpendicular to" on lines is
   (a) reflexive (b) equivalence (c) transitive (d) symmetric only

7. $aRb \iff a \leq b$ on R is not
   (a) symmetric (b) reflexive (c) transitive (d) a relation

8. Equivalence classes of $aRb \iff 2 \mid (a-b)$ on $\mathbb{Z}$
   (a) 1 (b) 3 (c) 2 (d) infinitely many

9. Number of functions from $\{1,2,3\}$ to $\{a,b\}$
   (a) 6 (b) 5 (c) 9 (d) 8

10. One-one functions from a 4-set to a 3-set
    (a) 24 (b) 0 (c) 12 (d) 81

11. $f(x) = x^3$, $R \to R$
    (a) many-one (b) into (c) bijective (d) not a function

12. If $f(x) = 3x - 4$, $f^{-1}(x) =$
    (a) $\frac{x+4}{3}$ (b) $3x+4$ (c) $\frac{x-4}{3}$ (d) $4-3x$

13. $g \circ f(x)$ for $f = x+1$, $g = x^2$
    (a) x^2+1 (b) $(x+1)^2$ (c) x^2+2 (d) $2x$

14. $[x]$ on R is
    (a) onto (b) one-one (c) neither one-one nor onto (d) bijective

15. An onto function from $\{1,2,3\}$ to itself is always
    (a) constant (b) identity (c) many-one (d) one-one

16. Equivalence relations on $\{1,2,3\}$ containing $(1,2)$
    (a) 2 (b) 1 (c) 3 (d) 4

17. $\{(1,2),(2,1)\}$ on $\{1,2\}$ is
    (a) symmetric (b) transitive (c) reflexive (d) equivalence

18. $f(x) = \log_2 x$, $(0,\infty) \to R$
    (a) not onto (b) into (c) many-one (d) bijective

19. Range of $f(x) = \frac{x}{1+|x|}$
    (a) R (b) $(-1,1)$ (c) $[0,1)$ (d) $[-1,1]$

20. Relations from a 2-set to a 3-set
    (a) 6 (b) 8 (c) 64 (d) 9

1 (c) · 2 (a) · 3 (d) · 4 (b) · 5 (b) · 6 (d) · 7 (a) · 8 (c) · 9 (d) · 10 (b) · 11 (c) · 12 (a) · 13 (b) · 14 (c) · 15 (d) · 16 (a) · 17 (a) · 18 (d) · 19 (b) · 20 (c)

Assertion–Reason (6)

(a) Both true, R explains A. (b) Both true, R does not explain A. (c) A true, R false. (d) A false, R true.

1. A: $f(x) = x^2$, $R \to R$ is not one-one. R: $f(-2) = f(2)$.
2. A: $(1,1),(2,2),(3,3)$ on $1,2,3$ is an equivalence relation. R: every equivalence relation is reflexive.
3. A: $f(x) = e^x$, $R \to R$ is not onto. R: $e^x > 0$ for all x.
4. A: There are 8 relations on $\{1,2\}$ that are reflexive. R: reflexive relations on an n-set number 2^{n^2-n}.
5. A: $f : N \to N$, $f(x) = x^2$ is one-one. R: the square of a natural number is a natural number.
6. A: Every bijection has an inverse. R: a function is invertible if and only if it is one-one and onto.

Key: 1 (a) · 2 (b) · 3 (a) · 4 (d) · 5 (b) · 6 (a)

Answers to the practice set

A. 1. $2^9 = 512$ 2. equivalence 3. one-one 4. 24 5. co-domain 6. 5 7. not onto 8. $\frac{x-3}{2}$

B. 1. F 2. T 3. T 4. T 5. T 6. F (9) 7. T 8. F (range $[-1, 1]$)

C. 1. $(a, b) \in R \Rightarrow (b, a) \in R$ 2. $\{(1, 2)\}$ on $\{1, 2\}$ 3. $\{3, 7\}$ 4. no (range $[1, \infty)$) 5. 12 6. 5 7. $\{-1, 0, 1\}$ 8. when it is bijective 9. $2^6 = 64$ 10. no

D. 1. $7x_1 - 2 = 7x_2 - 2 \Rightarrow x_1 = x_2$; $x = \frac{y+2}{7}$ 2. not transitive: $(1, 2), (2, 3) \in R$, $1 - 3 = -2$ even 3. $f(x) \geq 4$, so 1, 2, 3 have no pre-image 4. $\{1, 4, 7\}, \{2, 5\}, \{3, 6\}$ 5. $\frac{x_1}{x_1+1} = \frac{x_2}{x_2+1} \Rightarrow x_1 = x_2$ 6. $g \circ f = |5|x| - 2|$, $f \circ g = |5x - 2|$ 7. $\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ 8. no: odd numbers are not images

E. 1. $a + a$ even; symmetric; $a + c = (a + b) + (b + c) - 2b$; $[0]$ = even integers 2. as Ex 1.2 Q10; inverse $x = \frac{2y-1}{y-1}$ 3. $a + b = b + a$; $a + d = b + c \Rightarrow c + b = d + a$; add $a + d = b + c$ and $c + f = d + e$: $a + f = b + e$ 4. $f(2) = f(\frac{1}{2}) = \frac{2}{5}$; $|f(x)| \leq \frac{1}{2}$ so 1 is not attained 5. $A \times A$ 6. as NCERT Example 12

F. 1(a) reflexive $(\sqrt{3})$; not symmetric: $(\sqrt{3}, 0)$ gives $2\sqrt{3}$, $(0, \sqrt{3})$ gives 0; not transitive: $(\sqrt{3}, 1), (1, 2\sqrt{3}) \in R$, $(\sqrt{3}, 2\sqrt{3}) \notin R$ (b) bijective; $f^{-1}(x) = \frac{3x+1}{2}$ 2(a) $\frac{x_1}{x_1+2} = \frac{x_2}{x_2+2} \Rightarrow x_1 = x_2$; range $[-1, \frac{1}{3}]$; $f^{-1}(y) = \frac{2y}{1-y}$ (b) 8; 0; 6

G. 1. yes: $\{(1, 1)\}$ on $\{1, 2\}$ 2. $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$, so distinct inputs give distinct outputs 3. yes: $g(x_1) = g(x_2) \Rightarrow f(g(x_1)) = f(g(x_2)) \Rightarrow x_1 = x_2$ 4. $2^6 = 64$

Cases. 1: (i) $3^4 = 81$ (ii) no: 4 students, 3 rooms (iii) choose 2 for r_1: ${}^4C_2 = 6$, the other two get r_2, r_3 in 2 ways: 12 (iv) yes. 2: (i) yes (ii) no: $f(x) = 0$ needs $x = \frac{7}{5}$ (iii) $f^{-1}(x) = \frac{x+7}{5}$ (iv) integers $\equiv 3 \pmod 5$, i.e. $\ldots, -2, 3, 8, \ldots$

Last-night summary

- Relation on A = subset of $A \times A$; 2^{n^2} relations; empty and universal relations are trivial.
- Reflexive (every (a, a)), symmetric ($(a, b) \Rightarrow (b, a)$), transitive ($(a, b), (b, c) \Rightarrow (a, c)$); all three = equivalence; classes partition the set.
- One-one: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ (or strictly monotonic); onto: solve $y = f(x)$ inside the domain; bijective = both $\iff$ invertible.
- Counting $m \to n$: functions n^m, one-one ${}^nP_m$, bijections $n!$ if $m = n$; relations 2^{mn}.
- $g \circ f(x) = g(f(x))$, not commutative; f^{-1}: solve $y = f(x)$ for x.