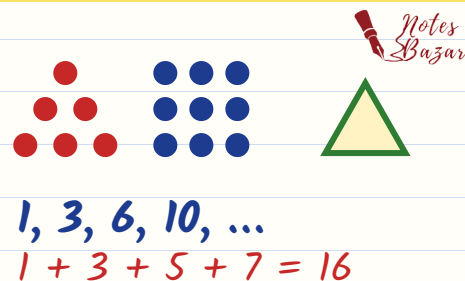


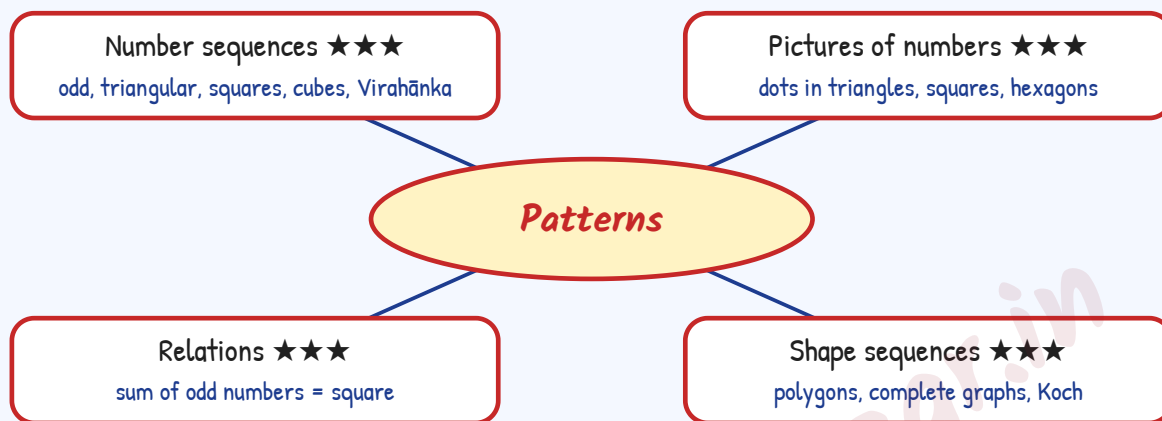
Chapter 1

Patterns in Mathematics

New NCERT Ganita Prakash, Grade 6 · 2026–27



Chapter map (★ = important for tests)



What is inside

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★ **Exam tip:** **Score here:** know the **rule** and the **next three terms** of every sequence in Table 1; be able to **draw** triangular, square and hexagonal numbers; remember $1 + 3 + 5 + \dots$ (**n odd numbers**) $= n \times n$ and $1 + 2 + \dots + n + \dots + 2 + 1 = n \times n$; count sides, lines and pieces in shape sequences.

Must-know: triangular 1, 3, 6, 10, 15 · squares 1, 4, 9, 16, 25 · cubes 1, 8, 27, 64, 125 · Virahāṅka 1, 2, 3, 5, 8, 13 · powers of 2: 1, 2, 4, 8, 16 · hexagonal 1, 7, 19, 37, 61.

1.1 What is mathematics? ★★

Definition 🖋️ **Mathematics** is, in large part, the **search for patterns**, and for the **explanations** of why those patterns exist. Patterns are all around us: in nature, our homes and schools, the motion of the sun, moon and stars, shopping, cooking, games, weather and technology. Mathematicians see it as both an **art** and a **science**. Explanations can be used far beyond where they were found: patterns in the motion of planets led to the theory of **gravitation** (satellites, rockets to the Moon and Mars); patterns in **genomes** help diagnose and cure diseases.

Figure it Out (page 2)

Ans. 1. Maths in everyday life: shopping (adding prices, checking change, comparing which pack is cheaper per kg); **cooking** (measuring, doubling a recipe); **time and travel** (clocks, calendars, fares, distances); **home** (measuring a room for tiles, sharing equally); **games and sport** (scores); **saving** (budgets).

Ans. 2. How maths propels humanity: **science** (measuring results, finding laws like gravitation); **economy and democracy** (banking, taxes, budgets, counting votes); **buildings and bridges** (geometry, loads); **machines** (phones, computers and TVs work with codes and algorithms; cars, trains and planes need speed and shape calculations); **calendars and clocks** (patterns of the Sun, Moon and Earth).

1.2 Patterns in numbers ★★★

Definition 🖋️ The most basic patterns are patterns of **whole numbers** 0, 1, 2, 3, 4, ... The branch of mathematics that studies them is **number theory**. **Number sequences** (lists of numbers following a rule) are among the most basic and fascinating patterns.

Table 1: sequence	Name
1, 1, 1, 1, 1, 1, ...	All 1's
1, 2, 3, 4, 5, 6, 7, ...	Counting numbers
1, 3, 5, 7, 9, 11, 13, ...	Odd numbers
2, 4, 6, 8, 10, 12, 14, ...	Even numbers
1, 3, 6, 10, 15, 21, 28, ...	Triangular numbers
1, 4, 9, 16, 25, 36, 49, ...	Squares
1, 8, 27, 64, 125, 216, ...	Cubes
1, 2, 3, 5, 8, 13, 21, ...	Virahāṅka numbers
1, 2, 4, 8, 16, 32, 64, ...	Powers of 2
1, 3, 9, 27, 81, 243, 729, ...	Powers of 3

Figure it Out (page 3)

Ans. 1. Can you recognise the pattern in each sequence? Yes: each one follows a rule, given in the table below.

2. Rewrite each sequence with the next three numbers and its rule.

Sequence	Next three	Rule (in words)
All 1's	1, 1, 1	every number is 1
Counting numbers	8, 9, 10	add 1 each time
Odd numbers	15, 17, 19	start at 1, add 2 each time
Even numbers	16, 18, 20	start at 2, add 2 each time
Triangular numbers	36, 45, 55	add 2, then 3, then 4, ... (one more each time); the 8th is $1 + 2 + \dots + 8$
Squares	64, 81, 100	$1 \times 1, 2 \times 2, 3 \times 3, \dots$: a number times itself
Cubes	343, 512, 729	$1 \times 1 \times 1, 2 \times 2 \times 2, 3 \times 3 \times 3, \dots$
Virahānka numbers	34, 55, 89	each number is the sum of the two before it ($13 + 21 = 34$)
Powers of 2	128, 256, 512	multiply by 2 each time
Powers of 3	2187, 6561, 19683	multiply by 3 each time

Definition 🖋️ **Virahānka numbers** are named after the Indian scholar **Virahānka**, who found them while counting rhythms in poetry; they are also known as **Fibonacci numbers**.

X Don't do this: Common mistake: confusing multiples of 2 (2, 4, 6, 8: add 2) with powers of 2 (1, 2, 4, 8, 16: multiply by 2).

★ **Exam tip:** How to find a rule: look at the **differences** between neighbours (odd: +2, +2; triangular: +2, +3, +4), or the **ratio** (powers of 3: $\times 3, \times 3$), or how a number comes from the **earlier** ones (Virahānka: add the two before).

Differences reveal the pattern: triangular $1 \rightarrow 3 \rightarrow 6 \rightarrow 10 \rightarrow 15 \rightarrow 21 \rightarrow 28$ differ by **2, 3, 4, 5, 6, 7**; squares $1 \rightarrow 4 \rightarrow 9 \rightarrow 16 \rightarrow 25$ differ by **3, 5, 7, 9**, the odd numbers!

1.3 Visualising number sequences ★★★

Definition Many sequences can be shown with **pictures** of dots or blocks. Visualising mathematical objects through pictures or diagrams is a very useful way to understand patterns.

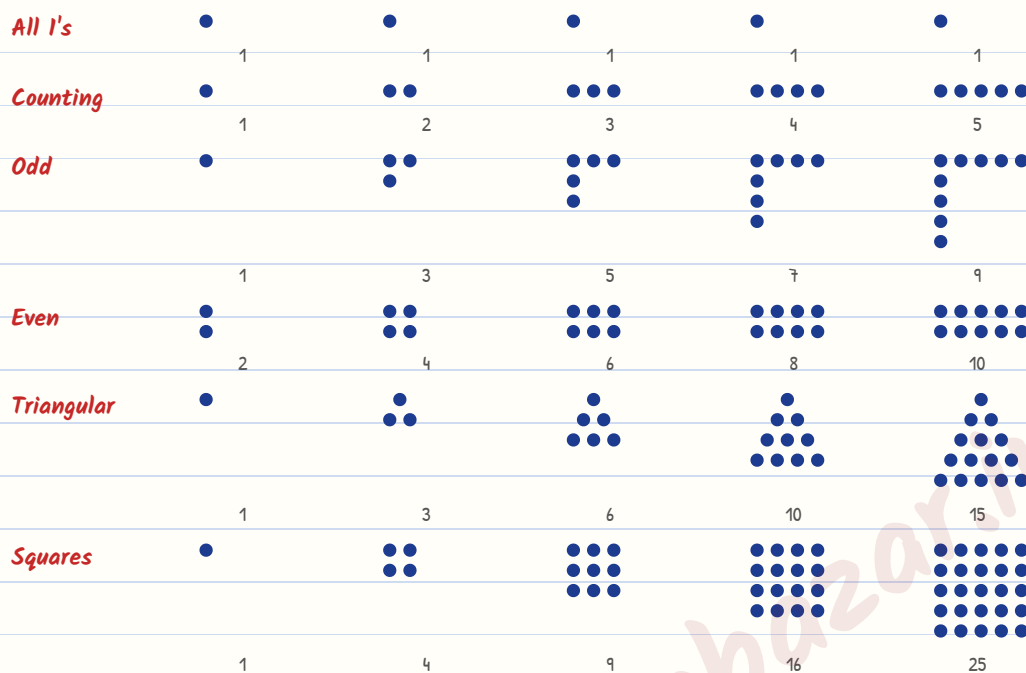


Table 2 (redrawn): pictures of the first five numbers of each sequence (cubes: 1, 8, 27, 64, 125 blocks stacked as cubes)

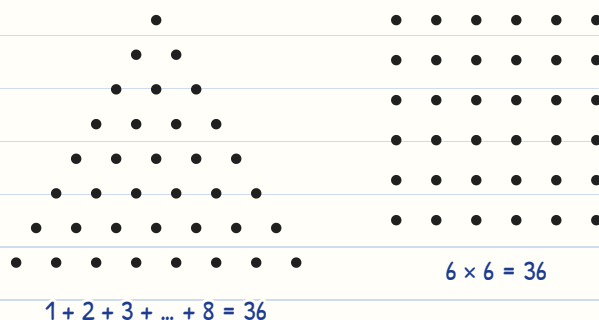
Figure it Out (page 5)

Ans. 1. The next (6th) picture: All 1's: 1 dot; counting: a row of 6; odd: 11 dots in an L (6 across, 6 down, sharing the corner); even: 12 in two rows of 6; triangular: 21 in 6 rows (1 to 6); squares: 36 in a 6×6 square; cubes: 216 blocks in a $6 \times 6 \times 6$ cube.

Ans. 2. Why the names? **Triangular**: that many dots make a **triangle** with rows of 1, 2, 3, 4, ... ($10 = 1 + 2 + 3 + 4$). **Squares**: they make a **square** with equal rows and columns ($16 = 4$ rows of 4). **Cubes**: that many small cubes stack into a bigger **cube** with equal length, breadth and height ($27 = 3 \times 3 \times 3$).

Ans. 3. 36 as a triangle and a square: $36 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$ (a triangle of 8 rows) $= 6 \times 6$ (a square).

Other numbers in different ways: **1** is a triangle, a square and a cube; **6** is a triangle ($1 + 2 + 3$) and a 2×3 rectangle; **10** is a triangle ($1 + 2 + 3 + 4$) and a 2×5 rectangle; **12** is a 3×4 or 2×6 rectangle.

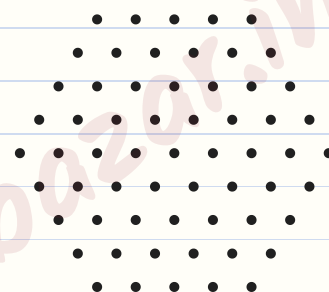


36 dots arranged as a triangle (8 rows) and as a 6×6 square

Ans. 4. Hexagonal numbers 1, 7, 19, 37: each picture adds a ring of dots around the previous hexagon; the rings have **6, 12, 18, 24, ...** dots (6 more each time):

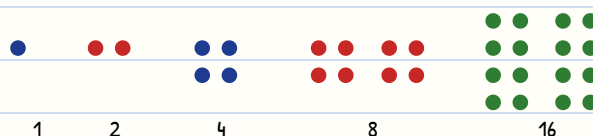
$1 \xrightarrow{+6} 7 \xrightarrow{+12} 19 \xrightarrow{+18} 37 \xrightarrow{+24} 61$.

The next number is 61.



The next hexagonal number: 61 dots ($37 + 24$)

Ans. 5. Pictures for powers of 2 and 3: each picture is **two (or three) copies** of the one before. Powers of 2: 1 dot $\rightarrow$ 2 in a row $\rightarrow$ a 2×2 square (4) $\rightarrow$ two squares (8, or a $2 \times 2 \times 2$ cube) $\rightarrow$ two of those (16). Powers of 3: 1 dot $\rightarrow$ 3 in a row $\rightarrow$ a 3×3 square (9) $\rightarrow$ three squares stacked into a $3 \times 3 \times 3$ cube (27) $\rightarrow$ three such cubes (81). Another way: a **branching tree** where every dot branches into 2 (or 3) new dots at the next level.



Powers of 2: each picture is two copies of the one before (1, 2, 4, 8, 16)

★ **Exam tip:** Key idea: the same number can be shown in different ways (36 as a triangle and as a square), depending on the context.

1.4 Relations among number sequences ★★★

Adding odd numbers gives squares

$$1 = 1$$

$$1 + 3 = 4$$

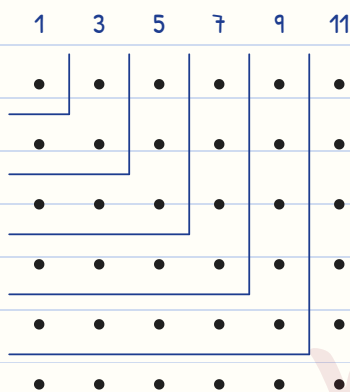
$$1 + 3 + 5 = 9$$

$$1 + 3 + 5 + 7 = 16$$

$$1 + 3 + 5 + 7 + 9 = 25$$

$$1 + 3 + 5 + 7 + 9 + 11 = 36$$

Definition  **Why?** A picture explains it: split a square grid of dots into **L-shaped layers**; the layers have 1, 3, 5, 7, ... dots. Such a picture can be made for a square of **any size**, so the pattern goes on **forever**. The **reason** a pattern happens is as important and exciting as the pattern itself.



$1 + 3 + 5 + 7 + 9 + 11 = 36$: each L-shaped layer adds the next odd number

Ans. Think: the first 10 odd numbers make a 10×10 square: $1 + 3 + 5 + \dots + 19 = 10 \times 10 = 100$.

The first 100 odd numbers make a 100×100 square: $1 + 3 + \dots + 199 = 100 \times 100 = 10000$.

Adding up and down also gives squares

$$1 = 1$$

$$1 + 2 + 1 = 4$$

$$1 + 2 + 3 + 2 + 1 = 9$$

$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$$

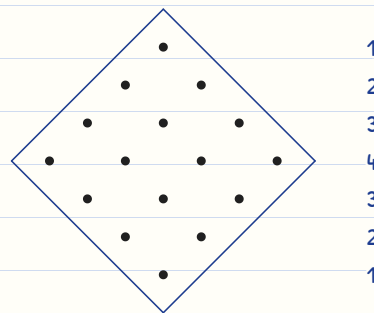
$$\dots \text{ up to 6 and back } = 36$$

★ **Exam tip:** Remember: sum of the first n odd numbers = $n \times n$. The last odd number in the sum tells you n : last number = $2 \times n - 1$ (for 19, $n = 10$; for 199, $n = 100$).

✗ **Don't do this:** Common mistake: " $1 + 3 + 5 + \dots + 19 = 19 \times 19$ ". No: 19 is the 10th odd number, so the sum is $10 \times 10 = 100$.

Figure it Out (page 8)

Ans. 1. Why up-and-down sums are squares: take a square of dots and count it along **slanting lines** (diagonals). The slanting rows have 1, 2, 3, ... up to the side, then back down to 1. A 4×4 square (16 dots) gives $1 + 2 + 3 + 4 + 3 + 2 + 1$. The same works for any size, so going up to n and back down gives $n \times n$.



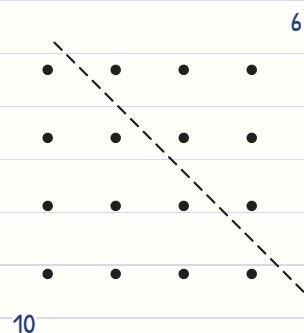
The 16 dots of a 4×4 square, turned on its corner, form rows of 1, 2, 3, 4, 3, 2, 1

Ans. 2. $1 + 2 + \dots + 99 + 100 + 99 + \dots + 2 + 1$ is the slanting count of a 100×100 square $= 100 \times 100 = 10000$.

Ans. 3. Adding the All 1's sequence: up: $1, 1 + 1, 1 + 1 + 1, \dots = 1, 2, 3, 4, \dots$ (the **counting numbers**). Up and down means taking 1, then 3, then 5, ... terms (as $1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1$ have 1, 3, 5 terms): $1, 1 + 1 + 1, 1 + 1 + 1 + 1 + 1, \dots = 1, 3, 5, 7, \dots$ (the **odd numbers**).

Ans. 4. Adding the counting numbers up: $1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, \dots = 1, 3, 6, 10, 15, \dots$, the **triangular numbers**. A smaller picture: rows of 1, 2, 3, 4, ... dots placed one below the other, each starting at the left edge, form a right-angled **staircase triangle**.

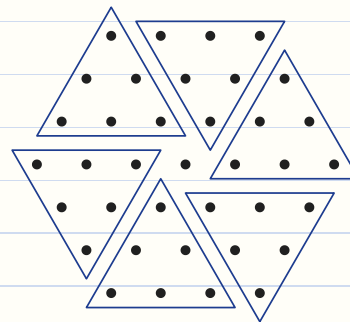
Ans. 5. Adding consecutive triangular numbers: $1 + 3 = 4, 3 + 6 = 9, 6 + 10 = 16, 10 + 15 = 25, \dots$: the **square numbers**. **Why:** cut a square of dots along a slanting line just above the diagonal into two staircase triangles, one with rows 1, 2, 3, 4 (10 dots) and one with rows 1, 2, 3 (6 dots): two consecutive triangular numbers.



A 4×4 square splits into triangles of 10 and 6 dots: $6 + 10 = 16$

Ans. 6. Adding powers of 2: $1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8, \dots = 1, 3, 7, 15, 31, 63, \dots$. Adding 1 to each gives $2, 4, 8, 16, 32, 64, \dots$, the powers of 2 again: each sum is **one less than the next power of 2**. **Why:** start with the extra 1 and keep adding: $1 + 1 = 2, 2 + 2 = 4, 4 + 4 = 8, 8 + 8 = 16, \dots$ each step **doubles** the total.

Ans. 7. $6 \times \text{triangular} + 1$: $6 \times 1 + 1 = 7, 6 \times 3 + 1 = 19, 6 \times 6 + 1 = 37, 6 \times 10 + 1 = 61, \dots$: the **hexagonal numbers** 7, 19, 37, 61, 91. **Picture:** a hexagon of dots is a **centre dot** plus **6 equal triangles** around it, each a triangular number of dots.



$37 = 6 \times 6 + 1$: six triangles of 6 dots around the centre dot

Ans. 8. Adding hexagonal numbers: $1, 1 + 7 = 8, 8 + 19 = 27, 27 + 37 = 64, \dots$: the **cubes**. **Picture of a cube:** build a cube from one corner. A $1 \times 1 \times 1$ cube is 1 block; to make $2 \times 2 \times 2$ add 7 blocks covering three faces; to make $3 \times 3 \times 3$ add 19 more on three faces, and so on. Each added three-faced shell, **seen from the corner**, looks like a hexagon and has a hexagonal number of blocks: $8 - 1 = 7, 27 - 8 = 19, 64 - 27 = 37$.

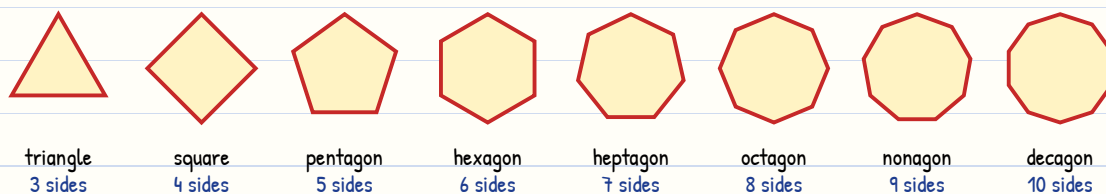
Ans. 9. Our own patterns (examples):

- **Even numbers** = $2 \times \text{counting numbers}$ (two equal rows of dots); also counting + counting: $1 + 1, 2 + 2, 3 + 3, \dots$
- **Sums of cubes are squares of triangular numbers:** $1 = 1 \times 1, 1 + 8 = 9 = 3 \times 3, 1 + 8 + 27 = 36 = 6 \times 6, 1 + 8 + 27 + 64 = 100 = 10 \times 10$ (1, 3, 6, 10 are triangular).
- **Sums of Virahāṅka numbers** are 2 less than a later Virahāṅka number: $1 + 2 + 3 + 5 = 11 = 13 - 2$.
- **Powers of 3:** $1 + 3 + 9 + 27 = 40$, and $2 \times 40 + 1 = 81$, the next power of 3.
- **Odd + odd = even**, and consecutive odd numbers differ by 2.

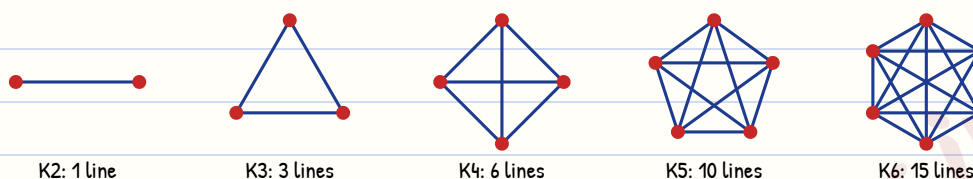
★ **Exam tip:** Exam tip: when asked "why", draw a **small picture** (dots or blocks) and say that the same picture works for **any size**.

1.5 Patterns in shapes ★★★

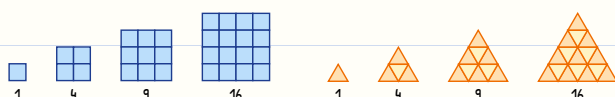
Definition Other basic patterns are patterns of **shapes**, in one, two or three dimensions (1D, 2D, 3D) or more. The branch of mathematics that studies them is **geometry**. **Shape sequences** are one important type of shape pattern (Table 3).



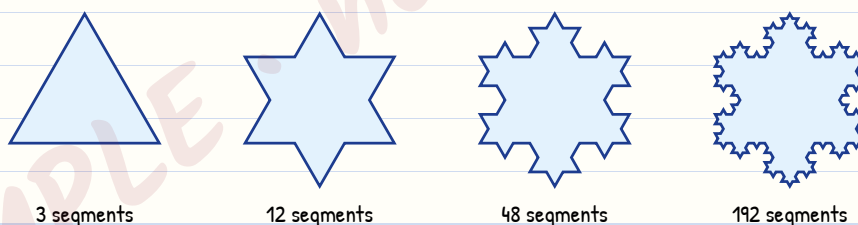
Regular polygons: 3, 4, 5, ... equal sides and equal corners (after Table 3)



Complete graphs: every point joined to every other point (1, 3, 6, 10, 15 lines)



Definition Regular means all **sides** are equal and all **angles** (corners) are equal. A **complete graph** K_n has n points with **every pair** joined by a line.



Koch snowflake: each segment is replaced by a "speed bump" of 4 smaller segments ($3 \times$ powers of 4)

Ans. Figure it Out (page 11), 1. Patterns in Table 3: **regular polygons** gain one side (and corner) each time; **complete graphs** join every point to every other; **stacked squares** add a row and a column each time ($n \times n$); **stacked triangles** add a row each time; in the **Koch snowflake** every segment becomes a speed bump of 4 smaller segments.

2. The next shapes: an 11-sided regular polygon (hendecagon); K_7 (7 points, 21 lines); a 6×6 stack (36 squares); 6 rows of triangles (36); the next Koch shape is **very hard to draw by hand**, as each of its many tiny segments needs a still tinier bump.

1.6 Relation to number sequences ★★★

Definition  Shape sequences are often related to number sequences: the regular polygons have 3, 4, 5, 6, 7, 8, 9, 10, ... sides, so they are called triangle, quadrilateral (square), pentagon, hexagon, heptagon, octagon, nonagon, decagon.

Figure it Out (pages 11–12)

Ans. 1. Sides and corners of regular polygons: sides 3, 4, 5, 6, ... (counting numbers from 3); corners **the same** 3, 4, 5, 6, ... **Why:** in a closed shape with straight sides, every side ends at a corner where it meets the next side, and each corner joins exactly two sides; going round, sides and corners alternate, so there are as many corners as sides.

Ans. 2. Lines in complete graphs: K_2, K_3, K_4, K_5, K_6 have 1, 3, 6, 10, 15 lines: the **triangular numbers**. **Why:** a new point must be joined to every point already there: $K_3 = 1 + 2$, $K_4 = 1 + 2 + 3$, $K_5 = 1 + 2 + 3 + 4$, and adding counting numbers gives triangular numbers.

Ans. 3. Stacked squares: 1, 4, 9, 16, 25 little squares, the **square numbers**: the n -th shape has n rows of n squares.

Ans. 4. Stacked triangles: 1, 4, 9, 16, 25 little triangles, again **squares**. Row by row from the top there are 1, 3, 5, 7, ... triangles (each row has one more upright and one more upside-down triangle), and $1 + 3 + 5 + \dots$ (n odd numbers) $= n \times n$.

Ans. 5. Koch snowflake segments: the triangle has 3 segments; each speed bump turns 1 segment into 4, so: 3, 12, 48, 192, 768, ... = **$3 \times$ powers of 4** (3×1 , 3×4 , 3×16 , 3×64 , 3×256).

Shape sequence	What we count	Number sequence
regular polygons	sides (= corners)	3, 4, 5, 6, ... (counting from 3)
complete graphs $K_2 \dots K_6$	lines	1, 3, 6, 10, 15 (triangular)
stacked squares	little squares	1, 4, 9, 16, 25 (squares)
stacked triangles	little triangles	1, 4, 9, 16, 25 (squares)
Koch snowflake	segments	3, 12, 48, 192 ($3 \times$ powers of 4)

Key patterns at a glance ★★★

Sequence	First terms	10th term	How to get the n -th term
odd numbers	1, 3, 5, 7, 9	19	$2 \times n - 1$
even numbers	2, 4, 6, 8, 10	20	$2 \times n$
triangular	1, 3, 6, 10, 15	55	$1 + 2 + 3 + \dots + n$
squares	1, 4, 9, 16, 25	100	$n \times n$
cubes	1, 8, 27, 64, 125	1000	$n \times n \times n$
Virahāṅka	1, 2, 3, 5, 8	89	sum of the two before
powers of 2	1, 2, 4, 8, 16	512	double the one before
hexagonal	1, 7, 19, 37, 61	271	$6 \times (\text{triangular before}) + 1$

Four big relations

$$1 + 3 + 5 + \dots (n \text{ odd}) = n \times n$$

$$1 + 2 + \dots + n + \dots + 2 + 1 = n \times n$$

$$\text{two consecutive triangular} = \text{square}$$

$$1 + 2 + 4 + \dots (\text{powers of 2}) = \text{next power} - 1$$

X Don't do this: Common mistakes: (1) counting K_n lines as n (K_4 has 6, not 4); (2) thinking triangular numbers go up by 2 (they go up by 2, 3, 4, ...); (3) writing the Virahāṅka rule as "add 2"; (4) calling 1, 2, 4, 8 "even numbers" (1 is odd; they are powers of 2).

Worked examples

Ans. E1. Is 100 triangular? Triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105. 100 is **not** in the list (it lies between 91 and 105). It **is** a square (10×10).

E2. Sum $1 + 3 + 5 + \dots + 49$? $49 = 2 \times 25 - 1$, so there are 25 odd numbers: $25 \times 25 = 625$.

Ans. E3. Next two Virahāṅka numbers after 34, 55: $34 + 55 = 89$, $55 + 89 = 144$.

E4. Lines in K_8 : $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$.

E5. $1 + 2 + 4 + 8 + 16 + 32 = 64 - 1 = 63$.

★ **Exam tip:** Exam tip: show your rule in words **and** check it on two terms before writing the next ones.

Exam-style questions *Exam-style*

1. Write the next two triangular numbers after 28. [1]
Ans. 36, 45. [2]
2. Which branch of maths studies patterns in shapes? [1]
Ans. Geometry. [3]
3. Find $1 + 3 + 5 + \dots + 29$. [2]
Ans. 29 is the 15th odd number: $15 \times 15 = 225$. [3]
4. How many lines does K_7 have? [2]
Ans. $1 + 2 + \dots + 6 = 21$. [3]
5. Show with dots that 15 is triangular. [2]
Ans. Rows of 1, 2, 3, 4, 5 dots. [3]
6. Explain why sums of odd numbers are squares. [3]
Ans. L-shaped layers of a square have 1, 3, 5, ... dots. [3]
7. Write the Koch snowflake segments for 4 shapes and the rule. [3]
Ans. 3, 12, 48, 192; multiply by 4. [3]
8. Give five sequences from Table 1 with their next three terms and rules. [5]
Ans. E.g. odd (15, 17, 19; +2), triangular (36, 45, 55; +2, +3, ...), squares (64, 81, 100), Virahāṅka (34, 55, 89; add two before), powers of 2 (128, 256, 512; $\times 2$). [5]

Case study 1: the stadium seats Exam-style. A small stand has 1 seat in the top row, 2 in the next, 3 in the next, and so on.

- (i) Seats with 4 rows? **10**. [1]
- (ii) With 8 rows? **36**. [1]
- (iii) What are these numbers called? **Triangular numbers**. [1]
- (iv) The seats of a 4-row stand and a 3-row stand together are rearranged in equal rows. What shape and how many?
 $10 + 6 = 16$ seats, a 4×4 square. [2]

Case study 2: handshakes Exam-style. At a meeting every person shakes hands once with every other person.

- (i) Handshakes for 3 people? **3**. [1]
- (ii) For 5 people? **10**. [1]
- (iii) Which shape sequence shows this? **Complete graphs**. [1]
- (iv) For 10 people? **$1 + 2 + \dots + 9 = 45$** . [2]

Practice set: test yourself *Exam-style*

Try every question before looking at the answers on page 16. 40 or more out of 48 means you are ready for the test.

A. Fill in the blanks (1 mark each)

1. The study of patterns in whole numbers is _____.
2. The 7th odd number is _____.
3. The 6th triangular number is _____.
4. $7 \times 7 \times 7 =$ _____, a cube.
5. After 21, the next Virahānka number is _____.
6. $2 \times 2 \times 2 \times 2 \times 2 =$ _____.
7. A regular nonagon has _____ sides.
8. K_5 has _____ lines.

B. True or false? (1 mark each)

1. 36 is both triangular and square.
2. $1 + 3 + 5 + 7 + 9 = 25$.
3. Powers of 2 are the same as even numbers.
4. The next hexagonal number after 37 is 61.
5. A regular polygon can have unequal sides.
6. Adding consecutive triangular numbers gives cubes.
7. The 3rd Koch shape has 48 segments.
8. Stacked triangles give square numbers.

C. Very short answer (1 mark each)

1. Write the first five cubes.
2. What is $1 + 2 + 3 + 4 + 3 + 2 + 1$?
3. Write the next term: 1, 3, 9, 27, ...
4. Name a shape with 7 sides.
5. Which sequence do we get by adding the All 1's up?

6. Is 50 a square number?
7. Find $1 + 2 + 4 + 8$.
8. Name the sequence 1, 7, 19, 37.
9. How many corners has a decagon?
10. What is $6 \times 10 + 1$?

D. Short answer I (2 marks each)

1. Write the rule and next two terms: 2, 4, 6, 8.
2. Find $1 + 3 + 5 + \dots + 39$.
3. Draw 10 as a triangle of dots.
4. Why is 16 called a square number?

5. Write the next three Virahānka numbers after 13.
6. Find the sum of the first 20 odd numbers.
7. How many little squares in a 7×7 stack?
8. Find $1 + 2 + \dots + 50 + \dots + 2 + 1$.

E. Short answer II (3 marks each)

1. Show with a picture that $6 + 10 = 16$.
2. Explain why hexagonal numbers are $6 \times \text{triangular} + 1$.
3. Find the lines in K_2 to K_6 and explain the pattern.
4. Find 1, $1 + 2$, $1 + 2 + 4$, ... for six terms and add 1 to each.
5. Write the first six square numbers and the differences between them.
6. Explain why a polygon has as many corners as sides.

F. Long answer (5 marks each)

1. Draw pictures of the first four triangular, square and hexagonal numbers.
2. Explain three ways of getting square numbers from other sequences.
3. Describe five shape sequences and their number sequences.
4. "Mathematics is the search for patterns and their explanations." Explain with examples.

G. Challenge (HOTS) questions

1. Is 1000 a triangular number? (Hint: the 44th triangular number is 990.)
2. What is $1 + 8 + 27 + 64 + 125$? Is it a square?
3. Find a number between 100 and 200 that is both a square and a Virahanka number.
4. How many segments has the 6th Koch shape?

Case study 3: tiles on a floor Exam-style. Rani lays square tiles in L-shaped bands around one corner tile: 1 tile, then a band of 3, then a band of 5, and so on.

(i) Tiles after 4 bands? [1] (ii) Tiles in the 6th band? [1] (iii) Tiles after 10 bands? [1] (iv) What shape is always formed, and why? [2]

My score (fill in after checking the answers)

Section	A / 8	B / 8	C / 10	D / 16	E / 18	F / 20	G (bonus)	Total / 80
My marks								

Rough work

20 MCQs

1. The branch studying patterns in whole numbers is
(a) algebra (b) geometry (c) number theory (d) statistics
 2. The next odd number after 13 is
(a) 15 (b) 14 (c) 16 (d) 17
 3. 1, 3, 6, 10, 15 are
(a) squares (b) cubes (c) odd numbers (d) triangular numbers
 4. The 8th square number is
(a) 16 (b) 64 (c) 49 (d) 36
 5. $4 \times 4 \times 4$ equals
(a) 16 (b) 64 (c) 12 (d) 44
 6. In Virahānka numbers, after 8 and 13 comes
(a) 18 (b) 104 (c) 26 (d) 21
 7. The next power of 2 after 64 is
(a) 128 (b) 96 (c) 66 (d) 256
 8. $1 + 3 + 5 + 7 + 9 + 11$ equals
(a) 30 (b) 32 (c) 36 (d) 34
 9. The sum of the first 100 odd numbers is
(a) 5,050 (b) 100 (c) 1,000 (d) 10,000
 10. $1 + 2 + 3 + 2 + 1$ equals
(a) 6 (b) 9 (c) 8 (d) 10
 11. The next hexagonal number after 37 is
(a) 55 (b) 59 (c) 61 (d) 74
 12. Adding hexagonal numbers $1 + 7 + 19$ gives
(a) 27 (b) 26 (c) 36 (d) 25
 13. $3 + 6$ equals a
(a) cube (b) square number (c) triangular number (d) power of 2
 14. The number of lines in K_6 is
(a) 6 (b) 12 (c) 15 (d) 30
 15. A regular octagon has
(a) 6 sides (b) 7 sides (c) 9 sides (d) 8 sides
 16. Stacked squares give
(a) square numbers (b) cubes (c) odd numbers (d) triangular numbers
 17. The Koch snowflake segment counts are
(a) 3, 12, 48 (b) 3, 6, 9 (c) 3, 9, 27 (d) 3, 6, 12
 18. $1 + 2 + 4 + 8 + 16$ equals
(a) 30 (b) 33 (c) 32 (d) 31
 19. The study of shape patterns is
(a) number theory (b) geometry (c) arithmetic (d) data handling
 20. A number that is both triangular and square is
(a) 10 (b) 16 (c) 36 (d) 25
- 1 (c) · 2 (a) · 3 (d) · 4 (b) · 5 (b) · 6 (d) · 7 (a) · 8 (c) · 9 (d) · 10 (b) · 11 (c) · 12 (a) · 13 (b) · 14 (c) · 15 (d) · 16 (a) · 17 (a) · 18 (d) · 19 (b) · 20 (c)

Assertion-Reason (6)

(a) Both true, R explains A. (b) Both true, R does not explain A. (c) A true, R false. (d) A false, R true.

1. A: $1 + 3 + 5 + 7 = 16$. R: The L-shaped layers of a 4×4 square have 1, 3, 5, 7 dots.
2. A: K_5 has 10 lines. R: A new point joins every point already there.
3. A: 1, 2, 4, 8 are even numbers. R: Powers of 2 are found by doubling.
4. A: A regular hexagon has 6 corners. R: In a polygon each corner joins two sides.
5. A: 36 is a square number. R: The 8th triangular number is 36.
6. A: Stacked triangles give cubes. R: Their rows have 1, 3, 5, 7 triangles.

Key: 1 (a) · 2 (a) · 3 (d) · 4 (a) · 5 (b) · 6 (d)

Answers to the practice set

A. 1. number theory 2. 13 3. 21 4. 343 5. 34 6. 64 7. 9 8. 10

B. 1. T 2. T 3. F (1 is odd; powers double) 4. T 5. F 6. F (squares) 7. T 8. T

C. 1. 1, 8, 27, 64, 125 2. 16 3. 81 4. heptagon 5. counting numbers 6. no (between 49 and 64) 7. 15 8. hexagonal numbers 9. 10 10. 61

D. 1. add 2; 10, 12

2. 39 is the 20th odd number: $20 \times 20 = 400$

3. rows of 1, 2, 3, 4 dots

4. 16 dots make a 4×4 square

5. 21, 34, 55

6. $20 \times 20 = 400$

7. 49

8. $50 \times 50 = 2500$

E. 1. cut a 4×4 square of dots just above the diagonal: triangles of 10 and 6

2. a hexagon of dots = centre dot + 6 equal triangles (e.g. $37 = 6 \times 6 + 1$)

3. 1, 3, 6, 10, 15; each new point adds as many lines as points already there

4. 1, 3, 7, 15, 31, 63; adding 1 gives 2, 4, 8, 16, 32, 64

5. 1, 4, 9, 16, 25, 36; differences 3, 5, 7, 9, 11 (odd numbers)

6. each side ends at a corner shared with the next side

F. 1. pp. 4–5 2. odd sums, up-and-down sums, consecutive triangular sums (pp. 6–7) 3. table on p. 10 4. p. 2 and the relations

G. 1. no: the 45th triangular number is $990 + 45 = 1035$, so 1000 is skipped 2. $225 = 15 \times 15$, yes (and 15 is triangular) 3. $144 = 12 \times 12$ (a Virahāṅka number: $55 + 89$) 4. $3 \times 4 \times 4 \times 4 \times 4 \times 4 = 3072$

Case study 3. (i) $1 + 3 + 5 + 7 = 16$ (ii) 11 (iii) 100 (iv) always a square: n L-shaped bands of odd numbers of tiles make an $n \times n$ square

Last-night summary

- Mathematics = the search for patterns and for why they exist; number theory studies number patterns; geometry studies shape patterns.
- Table 1: all 1's, counting, odd, even, triangular (1, 3, 6, 10, 15), squares, cubes, Virahāṅka (add two before), powers of 2 and 3.
- Pictures: triangles, squares and cubes of dots and blocks; 36 is triangular and square; hexagonal numbers 1, 7, 19, 37, 61.
- Relations: odd sums = squares ($n \times n$); up-and-down sums = squares; two consecutive triangular = square; $6 \times \text{triangular} + 1 = \text{hexagonal}$; hexagonal sums = cubes; $1 + 2 + 4 + \dots = \text{next power of } 2 - 1$.
- Shapes: regular polygons (3, 4, 5 sides...), complete graphs (1, 3, 6, 10, 15 lines), stacked squares and triangles (squares), Koch snowflake (3, 12, 48, 192).