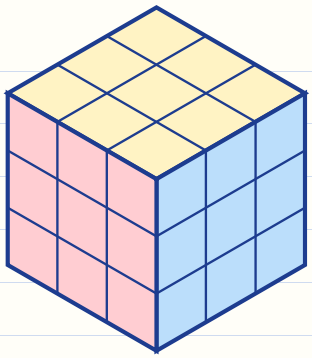
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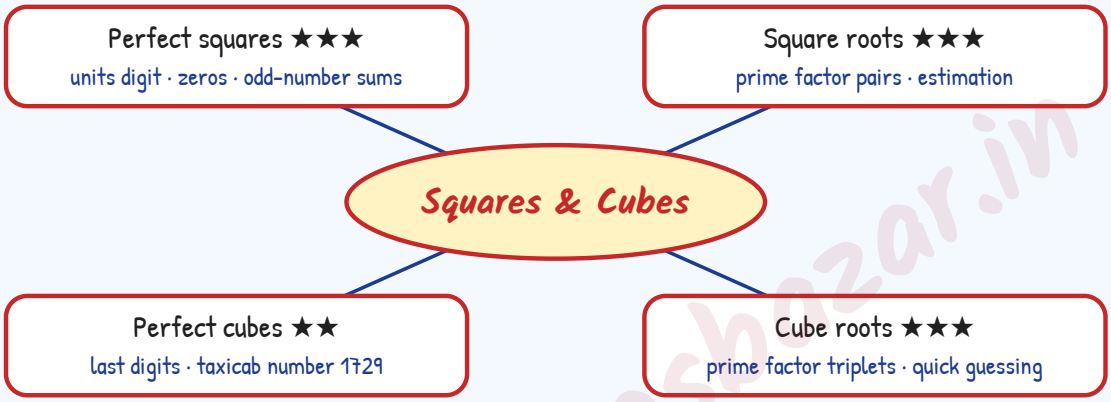
Chapter 1 A Square and a Cube

New NCERT Ganita Prakash, Grade 8 Part 1 · 2026-27



$3 \times 3 \times 3 = 27$

Chapter map (★ = important for tests)



What is inside

1. The locker puzzle and why squares have an odd number of factors
2. Squares: patterns, odd numbers, triangular numbers, square roots
3. All 9 questions of Figure it Out (page 10) solved
4. Cubes, taxicab numbers, cube roots, successive differences
5. All 5 questions of Figure it Out (page 16) and the Square Pairs puzzle
6. Exam-style questions and 3 case studies (p. 12, 14)
7. Practice set: 44 graded questions + 4 HOTS, 20 MCQs, 6 A–R, full answers, formula sheet

★ **Exam tip:** **Score here**: almost every test asks "is it a perfect square / cube?" and "find the smallest number to multiply or divide by". Always **write the prime factorisation**, then **ring the pairs** (for squares) or **triplets** (for cubes). Whatever is left over is your answer.

Must-know: squares 1^2 to 30^2, cubes 1^3 to 20^3 (tables and 9).

Based on NCERT "Ganita Prakash" Grade 8 Part 1 (2026-27), Chapter 1. Figures redrawn and solutions written by us. New book: questions are Exam-style.

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The locker puzzle ★★

Queen Ratnamanjuri's will: 100 people and 100 closed lockers. Person 1 opens every locker; person 2 **toggles** (opens if closed, closes if open) every 2nd locker; person 3 every 3rd, and so on up to person 100. Which lockers stay open? Khoisnam knew the answer **before** anyone started.

Key idea: locker number n is toggled once by every person whose number is a **factor** of n. Locker 6 is touched by persons 1, 2, 3, 6: open, close, open, close. So

number of toggles = number of factors of the locker number

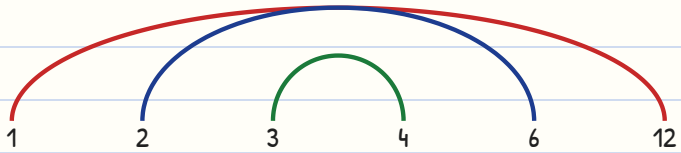
Odd number of toggles ⇒ **open**. Even number ⇒ **closed**.

> ***Definition*** **Partner factors**: factors of n come in pairs whose product is n. For 12: 1 × 12, 2 × 6, 3 × 4. Each pair adds **two** factors, so the total is usually **even**.

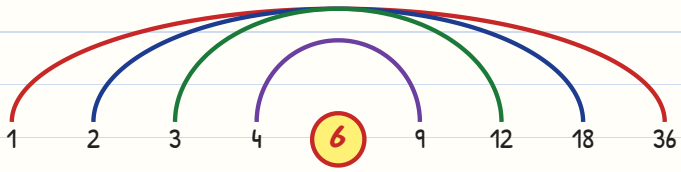
The exception: a pair made of the **same number twice**, such as 6 × 6 for 36. That factor counts only once, so the total becomes **odd**.

1	2	3	**4**	5	6	7	8	**9**	10
11	12	13	14	15	**16**	17	18	19	20
21	22	23	24	**25**	26	27	28	29	30
31	32	33	34	35	**36**	37	38	39	40
41	42	43	44	45	46	47	48	**49**	50
51	52	53	54	55	56	57	58	59	60
61	62	63	**64**	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	**100**

open at the end: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 (yellow)



12: 6 factors (even)



36: 9 factors (odd)

★ ***Exam tip:*** Only **square numbers** have an odd number of factors (they have a factor f with f × f = n). So the open lockers are exactly the squares: **1, 4, 9, 16, 25, 36, 49, 64, 81, 100**.

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Ans. The passcode: "the first five lockers touched exactly twice". Exactly two toggles = exactly two factors (1 and itself) = a prime number. Code: 2-3-5-7-11 (light blue above).

Ans. Think: does every number have an even number of factors? No. 1 (factor 1), 4 (1, 2, 4) and 9 (1, 3, 9) have an odd number. Does 36? Factors 1 & 36, 2 & 18, 3 & 12, 4 & 9, and 6 alone: 9 factors, odd. Every factor except 6 has a different partner.

1.1 Square numbers ★★★

Definition ✎ A number obtained by multiplying a number by itself is a square number (or a square). We write $n \times n = n^2$, read "n squared". The squares of natural numbers, 1, 4, 9, 16, 25, ..., are the perfect squares.

Why "square"? A square of side n units is made of n × n unit squares: its area is n^2 square units.

Side (units)	1	2	3	4	5	10
Area (sq units)	1	4	9	16	25	100

Ans. Think: a square of side 3/5 or 2.5 units? Yes, sides need not be whole numbers: area $\left(\frac{3}{5}\right)^2 = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$ sq units and $(2.5)^2 = 6.25$ sq units. (But 9/25 and 6.25 are not perfect squares: perfect squares are squares of natural numbers.)

Table of squares (learn it!)

n	n^2	n	n^2	n	n^2
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	26	676
7	49	17	289	27	729
8	64	18	324	28	784
9	81	19	361	29	841
10	100	20	400	30	900

Patterns in the table (Math Talk)

Number ends in	0	1 or 9	2 or 8	3 or 7	4 or 6	5
Its square ends in	0	1	4	9	6	5 (in fact 25)

- Squares end only in 0, 1, 4, 5, 6 or 9. None ends in 2, 3, 7 or 8.
- The units digits of 1^2, 2^2, ..., 10^2 go 1, 4, 9, 6, 5, 6, 9, 4, 1, 0: a mirror pattern, then it repeats.
- Consecutive squares differ by consecutive odd numbers: 4 − 1 = 3, 9 − 4 = 5, 16 − 9 = 7, ... (page 5).

Bonus shortcut for numbers ending in 5: $(n5)^2 = n \times (n + 1)$, followed by 25. $35^2 \rightarrow 3 \times 4 = 12 \rightarrow 1225$;

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$65^2 \longrightarrow 6 \times 7 = 42 \longrightarrow 4225$; $115^2 \longrightarrow 11 \times 12 = 132 \longrightarrow 13225$.

Testing with the units digit ★★★

✗ **Don't do this:** "Ends in 0, 1, 4, 5, 6 or 9, so it is a square." **Wrong!** 16 and 36 are squares ending in 6, but **26** is not. The units digit can only tell us when a number is **not** a square.

ends in 2, 3, 7 or 8 ⇒ **definitely not** a perfect square

Ans. **Think: 5 numbers that are not squares, from the units digit alone**: 42, 153, 297, 1008, 2222 (they end in 2, 3, 7, 8, 2).

Ans. **Think: the next two squares ending in 1 after 29^2**: only numbers ending in 1 or 9 have squares ending in 1: $31^2 = 961$ and $39^2 = 1521$.

Ans. **Think: which of 38^2, 34^2, 46^2, 56^2, 74^2, 82^2 end in 6?** A square ends in 6 only when the number ends in 4 or 6: $34^2 = 1156$, $46^2 = 2116$, $56^2 = 3136$, $74^2 = 5476$. ($38^2 = 1444$ and $82^2 = 6724$ end in 4.)

Zeros at the end ★★

Number	zeros	Square	zeros
10, 20, 40	1	100, 400, 1600	2
100, 200, 700, 900	2	10000, 40000, 490000, 810000	4
1000, 3000	3	1000000, 9000000	6

Ans. **Think: a number ends in 3 zeros. Its square? 6 zeros**: each end zero is a factor 10, and squaring doubles every factor. So a square always ends in an **even** number of zeros (0, 2, 4, …). **Can 1000 or 4000 be a square?** No: 3 zeros.

Parity of a number and its square ★

Ans. even × even = even and odd × odd = odd. So the **square has the same parity** as the number: $14^2 = 196$ (even), $15^2 = 225$ (odd). A square that is even has an even square root.

★ **Exam tip:** Combine the tests: 2468 ends in 8: not a square. 9000 ends in 3 zeros: not a square. 7225 ends in 25 and $72 = 8 \times 9$: so $7225 = 85^2$. (Shortcut from page 3, read backwards!)

Quick tests in one place: a number is **not** a perfect square if it (1) ends in 2, 3, 7 or 8, or (2) ends in an odd number of zeros, or (3) cannot be written as 1 + 3 + 5 + … (page 5), or (4) has a prime factor without a partner (page 6). Tests 1–2 can only say "no"; tests 3–4 give a sure answer either way.

Perfect squares and odd numbers ★★★

Differences of consecutive squares: $4 - 1 = 3$, $9 - 4 = 5$, $16 - 9 = 7$, $25 - 16 = 9$, … So adding odd numbers from 1 builds the squares:

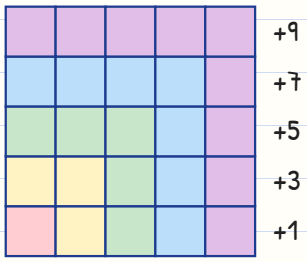
$1 = 1 \cdot 1 + 3 = 4 \cdot 1 + 3 + 5 = 9 \cdot 1 + 3 + 5 + 7 = 16$

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sum of the first n odd numbers = n^2

Visual proof (no words needed): each new inverted "L" around an n × n square adds n + n + 1 = **2n + 1** squares, the next odd number.

the n-th odd number = 2n − 1



1 + 3 + … + 9 = 5^2

every colour band is an "L" of an odd number of squares

> **Ans.** **Worked: find 36^2, given $35^2 = 1225$.** 1225 is the sum of the first 35 odd numbers. The 36th odd number is 2 × 36 − 1 = 71. So $36^2 = 1225 + 71 = 1296$. In general: $(n+1)^2 = n^2 + (2n+1)$, i.e. $(n+1)^2 = n^2 + n + (n+1)$.

> **Ans.** **Square test by subtracting odd numbers**: 25 − 1 = 24, − 3 = 21, − 5 = 16, − 7 = 9, − 9 = **0**. Five odd numbers reach 0 exactly, so $25 = 5^2$. For 38: 37, 34, 29, 22, 13, 2, then 2 − 13 = −11: we jump past 0, so **38 is not a square**.

> **Ans.** **Think: how many numbers lie between consecutive squares?** Between n^2 and $(n+1)^2$ there are (2n + 1) − 1 = **2n** numbers. Between 1 and 4: 2 (2, 3); between 4 and 9: 4; between 9 and 16: 6. The count goes up by 2 each time.

Squares in each block of 100 (Think)

1–100	101–200	201–300	301–400	401–500	501–600	601–700	701–800	801–900	901–1000
10	4	3	3	2	2	2	2	2	1

The squares spread out because the gaps (odd numbers) keep growing. The **largest square less than 1000 is $31^2 = 961$** ($32^2 = 1024$).

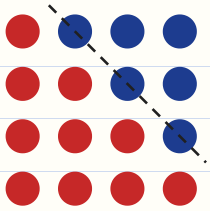
Squares and triangular numbers ★★

Triangular numbers: 1, 3, 6, 10, 15, 21, … (1, 1 + 2, 1 + 2 + 3, …). **Any two consecutive triangular numbers add up to a square**:

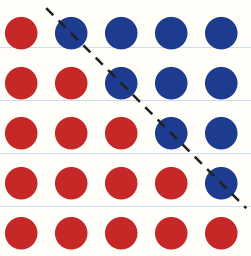
$1 + 3 = 4 = 2^2 \cdot 3 + 6 = 9 = 3^2 \cdot 6 + 10 = 16 = 4^2 \cdot 10 + 15 = 25 = 5^2$

Why: cut an n × n square of dots along the line just above the diagonal: you get two triangles with n and n − 1 rows.

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$6 + 10 = 16 = 4^2$



$10 + 15 = 25 = 5^2$

Square roots ★★★

Definition ✎ If $y = x^2$, then x is a **square root** of y. A square of area 49 cm² has side 7 cm, because $7^2 = 49$. Since $8^2 = 64$ **and** $(-8)^2 = 64$, every perfect square has **two** integer square roots, + and −: $\sqrt{64} = \pm 8$. In this chapter we use only the **positive** square root: $\sqrt{49} = 7$, $\sqrt{100} = 10$.

Three ways to find √n (is 576 a square?)

Method	How	Verdict
1. List squares	$20^2 = 400$, $21^2 = 441$, $22^2 = 484$, $23^2 = 529$, **$24^2 = 576$**	fine for small numbers, slow for big ones
2. Subtract odd numbers	$81 - 1 - 3 - 5 - 7 - 9 - 11 - 13 - 15 - 17 = 0$ in **9** steps: $\sqrt{81} = 9$	always works, but $\sqrt{729}$ would take 27 steps
3. Prime factorisation ★	split the prime factors into **two identical groups** (= make pairs)	best method for tests

Ans. Is 324 a square? $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 = (2 \times 2) \times (3 \times 3) \times (3 \times 3)$: all in pairs ✓. One from each pair: $2 \times 3 \times 3 = 18$. So $324 = (2 \times 3 \times 3) \times (2 \times 3 \times 3) = 18^2$, **$\sqrt{324} = 18$**.

Ans. Is 156 a square? $156 = (2 \times 2) \times 3 \times 13$. The 3 and the 13 have no partner, so **156 is not a perfect square**.

Ans. Think: 1156? $1156 = (2 \times 2) \times (17 \times 17)$ ✓. $\sqrt{1156} = 2 \times 17 = 34$.

Ans. Think: 2800? $2800 = (2 \times 2) \times (2 \times 2) \times (5 \times 5) \times 7$: the 7 is alone, so **not a square**.

Estimating square roots ★★

Ans. $\sqrt{1936}$: (i) $40^2 = 1600 < 1936 < 2500 = 50^2$, so the root is between 40 and 50. (ii) 1936 ends in 6, so the root ends in 4 or 6: **44 or 46**. (iii) $45^2 = (40 + 5)^2 = 1600 + 400 + 25 = 2025 > 1936$, so the root is below 45. (iv) Hence $\sqrt{1936} = 44$ (check: $44^2 = 1936$ ✓).

Ans. Aribam and Bijou: $\sqrt{250}$: $100 < 250 < 400$ gives $10 < \sqrt{250} < 20$. Closer: $15^2 = 225 < 250 < 256 = 16^2$. So $15 < \sqrt{250} < 16$, and since 250 is much nearer 256, **$\sqrt{250} \approx 16$** (a little less than 16; in fact about

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15.8).

Ans. Akhil's handkerchief: the cloth is a square of area 125 cm², so its side is √125 cm. A 15 cm square needs 225 cm²: not possible. $11^2 = 121 < 125 < 144 = 12^2$, so the biggest square with a whole-number side is 11 cm.

★ **Exam tip:** For "nearest square" questions, write the two squares on either side: $n^2 < N < (n+1)^2$. Then √N lies between n and n + 1, and it is closer to the root of whichever square is nearer N.

Figure it Out (page 10)

1. Which of the following numbers are not perfect squares? (i) 2032 (ii) 2048 (iii) 1027 (iv) 1089

Ans. 2032 ends in 2, 2048 ends in 8 and 1027 ends in 7: a square never ends in 2, 3, 7 or 8. 1089 ends in 9, so test it: $1089 = 3 \times 3 \times 11 \times 11 = (3 \times 11)^2 = 33^2$. Not perfect squares: (i), (ii) and (iii).

2. Which one among 64^2, 108^2, 292^2, 36^2 has last digit 4?

Ans. A square ends in 4 when the number ends in 2 or 8 ($2^2 = 4$, $8^2 = 64$). So 108^2 and 292^2 (= 11664 and 85264). 64^2 and 36^2 end in 6 ($4^2 = 16$, $6^2 = 36$).

3. Given $125^2 = 15625$, what is 126^2? (i) 15625 + 126 (ii) 15625 + 262 (iii) 15625 + 253 (iv) 15625 + 251 (v) 15625 + 512

Ans. $126^2 = 125^2$ + (the 126th odd number) = 15625 + (2 × 126 − 1) = 15625 + 251. Also 125 + 126 = 251. Answer (iv): 126^2 = 15876.

4. Find the length of the side of a square whose area is 441 m².

Ans. $441 = 3 \times 3 \times 7 \times 7 = (3 \times 7)^2$. Side = √441 = 3 × 7 = 21 m.

5. Find the smallest square number that is divisible by each of 4, 9 and 10.

Ans. A number divisible by 4, 9 and 10 is a multiple of LCM(4, 9, 10) = 180. Now 180 = (2 × 2) × (3 × 3) × 5: the 5 needs a partner. Multiply by 5: 180 × 5 = 900 = 30^2. (Check: 900 ÷ 4 = 225, 900 ÷ 9 = 100, 900 ÷ 10 = 90 ✓.)

6. Find the smallest number by which 9408 must be multiplied so that the product is a perfect square. Find the square root of the product.

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
	7

division ladder for 9408

Ans. 9408 = (2 × 2) × (2 × 2) × (2 × 2) × 3 × (7 × 7). Only the 3 is unpaired. So multiply by 3:
9408 × 3 = 28224 = (2 × 2) × (2 × 2) × (2 × 2) × (3 × 3) × (7 × 7).
√28224 = 2 × 2 × 2 × 3 × 7 = 168.

If the question said "divided by", the answer would also be 3: 9408 ÷ 3 = 3136 = 56^2.

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7. How many numbers lie between the squares of (i) 16 and 17 (ii) 99 and 100?

> **Ans.** Between n^2 and $(n+1)^2$ there are $2n$ numbers. (i) Between $16^2 = 256$ and $17^2 = 289$: $2 \times 16 = 32$ numbers (257 to 288). (ii) Between $99^2 = 9801$ and $100^2 = 10000$: $2 \times 99 = 198$ numbers.

8. Fill in the missing numbers: $1^2 + 2^2 + 2^2 = 3^2$; $2^2 + 3^2 + 6^2 = 7^2$; $3^2 + 4^2 + 12^2 = 13^2$; $4^2 + 5^2 + 20^2 = (__)^2$; $9^2 + 10^2 + (__)^2 = (__)^2$

> **Ans.** Pattern: the third number is the product of the first two, and the answer is one more than that product: $1 \times 2 = 2 \longrightarrow 3$; $2 \times 3 = 6 \longrightarrow 7$; $3 \times 4 = 12 \longrightarrow 13$.
> $4^2 + 5^2 + 20^2 = 21^2$ ($16 + 25 + 400 = 441$ ✓)
> $9^2 + 10^2 + 90^2 = 91^2$ ($81 + 100 + 8100 = 8281$ ✓)
> Why it works: $n^2 + (n+1)^2 + (n(n+1))^2 = (n(n+1)+1)^2$ always.

9. How many tiny squares are there in the picture (the book's 9×9 pattern of blocks)? Write the prime factorisation of the number of tiny squares.

> **Ans.** The picture is a 9×9 arrangement of blocks (straight grids and tilted diamonds), and each block holds $5 \times 5 = 25$ tiny squares. Total $= 81 \times 25 = 2025$.
> $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5 = 3^4 \times 5^2 = (3 \times 3 \times 5)^2 = 45^2$. A perfect square, as the square picture suggests!

More worked examples ★★★

> **Ans.** Smallest number to divide 2352 by to get a square; find the root. $2352 = (2 \times 2) \times (2 \times 2) \times 3 \times (7 \times 7)$. Divide by 3: $784 = 28^2$. $\sqrt{784} = 2 \times 2 \times 7 = 28$.

> **Ans.** $\sqrt{(169/289)} = \sqrt{169} / \sqrt{289} = 13/17$. $\sqrt{2.25} = \sqrt{(225/100)} = 15/10 = 1.5$. (Root of a fraction = root of top ÷ root of bottom.)

> **Ans.** 2025 students stand in a square formation. How many rows? $\sqrt{2025} = 45$ rows of 45.

> **Ans.** Smallest and largest 4-digit squares: $31^2 = 961$ has 3 digits, so the smallest is $32^2 = 1024$; $100^2 = 10000$ has 5 digits, so the largest is $99^2 = 9801$.

> **Ans.** Is 7744 a perfect square? Use the units digit and estimation. $80^2 = 6400 < 7744 < 8100 = 90^2$. It ends in 4, so the root would end in 2 or 8: 82 or 88. $85^2 = 7225 < 7744$, so try 88: $88^2 = 7744$ ✓. $\sqrt{7744} = 88$.

> ✗ **Don't do this:** Common mistakes: (1) "multiply by 3" when the leftover factors are 3 and 5: you must multiply by **all** unpaired primes, $3 \times 5 = 15$. (2) Forgetting to give the square root when asked. (3) Saying 26 is a square because 6 is a "square ending". (4) Writing $\sqrt{64} = 32$ (half!) instead of 8.

Quick drill ★★ *(answers below; cover them first!)*

1. Is 4096 a perfect square? If yes, find its root.
2. Smallest number to multiply 1008 by to get a square; the root?
3. Smallest number to divide 6000 by to get a square; the root?
4. Find 103^2 using $(100 + 3)^2$.
5. Find 98^2 using $(100 - 2)^2$.
6. Find 45^2 with the "ends in 5" shortcut.
7. Units digit of 4793^3?

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8. $\sqrt[3]{(-343)}$ = ?

9. $\sqrt[3]{(27/125)}$ = ?

10. Smallest number to multiply 675 by to get a cube?

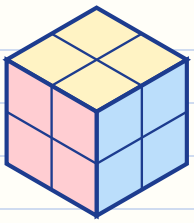
1. Yes: $4096 = 2^{12} = (2^6)^2$, $\sqrt{4096} = 64$ | 2. $1008 = 2^4 \times 3^2 \times 7$: $\times 7$, $\sqrt{7056} = 84$ | 3. $6000 = 2^4 \times 3 \times 5^3$: $\div 15$, $\sqrt{400} = 20$ | 4. $10000 + 600 + 9 = 10609$ | 5. $10000 - 400 + 4 = 9604$ | 6. $4 \times 5 = 20 \rightarrow 2025$ | 7. $3^3 = 27 \rightarrow 7$ | 8. -7 | 9. $3/5$ | 10. $675 = 3^3 \times 5^2$: $\times 5$, $3375 = 15^3$

1.2 Cubic numbers ★★★

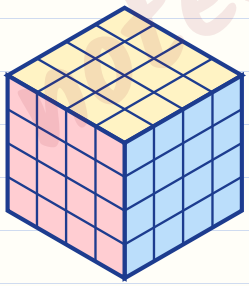
Definition ✎ A number obtained by multiplying a number by itself **three times** is a **cube**: $n \times n \times n = n^3$ ("n cubed"). 1, 8, 27, 64, 125, … are the **perfect cubes**.

Why "cube"? A cube of edge n is built from $n \times n \times n$ unit cubes: **n layers**, each of $n \times n$. Edge 2: **8** unit cubes; edge 3: **27**; edge 4: 4 layers of 16 = **64**.

Is 9 a cube? $2^3 = 8$ and $3^3 = 27$, so no number from **9 to 26** is a cube.



$2 \times 2 \times 2 = 8$



$4 \times 4 \times 4 = 64$

Table of cubes (learn it!)

n	n^3	n	n^3	n	n^3	n	n^3
1	1	6	216	11	1331	16	4096
2	8	7	343	12	1728	17	4913
3	27	8	512	13	2197	18	5832
4	64	9	729	14	2744	19	6859
5	125	10	1000	15	3375	20	8000

Last digit of a cube

n ends in	0	1	2	3	4	5	6	7	8	9
n^3 ends in	0	1	8	7	4	5	6	3	2	9

Unlike squares, a cube can end in **any** digit. 0, 1, 4, 5, 6, 9 keep their digit; **2 ↔ 8** and **3 ↔ 7** swap. Odd numbers have odd cubes, even numbers even cubes.

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Ans. Think: cubes with 1, 2 and 3 digits? 1 digit: 1, 8 (2). 2 digits: 27, 64 (2). 3 digits: 125, 216, 343, 512, 729 (5). Cubes thin out much faster than squares (there are 3, 6 and 22 squares with 1, 2 and 3 digits).

Ans. Think: can a cube end in exactly two zeros (00)? No. If a cube ends in 0, its cube root is a multiple of 10, so the cube is a multiple of $10^3 = 1000$ and ends in at least three zeros. A cube ends in 0, 3, 6, 9, ... zeros.

Cubes of fractions, decimals, negatives: $(4/6)^3 = 64/216$; $(13.08)^3 = 2237.810112$; $(-6)^3 = -6 \times -6 \times -6 = -216$. The cube of a negative number is **negative** (a square is always positive).

Taxicab numbers: Hardy's taxi was number 1729, "a rather dull number". Ramanujan: "No, it is the **smallest number that is a sum of two cubes in two different ways**": $1729 = 1^3 + 12^3 = 9^3 + 10^3$ (the **Hardy–Ramanujan number**).

Ans. Try this: the next taxicab numbers: $4104 = 2^3 + 16^3 = 9^3 + 15^3$ ($8 + 4096 = 729 + 3375$) and $13832 = 2^3 + 24^3 = 18^3 + 20^3$ ($8 + 13824 = 5832 + 8000$).

Cubes and consecutive odd numbers ★★

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

$31 + 33 + \ldots + 41 = 216 = 6^3$

Ans. Think: $91 + 93 + 95 + \ldots + 109$ without adding? Row n has **n** odd numbers and adds up to n^3. This row has 10 numbers, so it is row 10: the sum is $10^3 = 1000$. (Check: 10 numbers with average 100 give 1000 ✓.) Row 10 starts at 91 because rows 1–9 use $1 + 2 + \ldots + 9 = 45$ odd numbers, ending at 89.

Cube roots ★★★

Definition ✎ If $y = x^3$, then x is the **cube root** of y, written $x = \sqrt[3]{y}$. $\sqrt[3]{8} = 2$, $\sqrt[3]{27} = 3$, $\sqrt[3]{1000} = 10$; in general $\sqrt[3]{(n^3)} = n$.

Ans. Is 3375 a cube? $3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$. Three identical groups: $(3 \times 5) \times (3 \times 5) \times (3 \times 5) = 15^3$; or triplets: $(3 \times 3 \times 3) \times (5 \times 5 \times 5)$. $\sqrt[3]{3375} = 15$.

Ans. Is 500 a cube? $500 = (2 \times 2) \times (5 \times 5 \times 5)$. The 2s make only a pair, not a triplet: **500 is not a cube** (multiply by 2 to get $1000 = 10^3$).

Number	Its cube, prime factorised
$4 = 2 \times 2$	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
$6 = 2 \times 3$	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
$12 = 2 \times 2 \times 3$	$12^3 = 1728 = 2^3 \times 2^3 \times 3^3$

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Each prime factor of a number appears **three times** in its cube. So: **perfect cube ⇔ the prime factors can be grouped into triplets.**

> ***Ans.*** Find $\sqrt[3]{64}$, $\sqrt[3]{512}$, $\sqrt[3]{729}$. 64 = (2 × 2 × 2) × (2 × 2 × 2): **4**. 512 = 2^9 = three triplets of 2: 2 × 2 × 2 = **8**. 729 = 3^6 = (3 × 3 × 3) × (3 × 3 × 3): 3 × 3 = **9**.

Successive differences ★★

Squares	1	4	9	16	25	36
Level 1		3	5	7	9	11
Level 2			2	2	2	2

Cubes	1	8	27	64	125	216
Level 1		7	19	37	61	91
Level 2			12	18	24	30
Level 3				**6**	**6**	**6**

For squares the differences become equal (all 2) at **level 2**; for cubes they become equal (all 6) at **level 3**.

1.3 A pinch of history: the Babylonians listed squares and cubes on clay tablets around **1700 BCE**. In Sanskrit, **varga** meant a square (figure and power), **ghana** a cube, **varga-varga** the fourth power (used in India from at least the 3rd century BCE). A root was **mūla** (root of a plant): **varga-mūla** = square root, **ghana-mūla** = cube root; Arabic *jidhr* and Latin *radix* copied the idea. Āryabhaṭa (499 CE): "A square figure of four equal sides and the number representing its area are called varga." Brahmagupta (628 CE) used **pada** for root.

Figure it Out (page 16)

1. Find the cube roots of 27000 and 10648.

> ***Ans.*** 27000 = 27 × 1000 = (3 × 3 × 3) × (10 × 10 × 10): $\sqrt[3]{27000}$ = 3 × 10 = **30**.
>
> 10648 = 2 × 2 × 2 × 11 × 11 × 11 = (2 × 2 × 2) × (11 × 11 × 11): $\sqrt[3]{10648}$ = 2 × 11 = **22**.

2. What number will you multiply by 1323 to make it a cube number?

> ***Ans.*** 1323 = (3 × 3 × 3) × 7 × 7. The 7s form only a pair; one more 7 completes the triplet. Multiply by **7**: 1323 × 7 = 9261 = $(3 × 7)^3$ = **21^3**.

3. True or false? Explain. (i) The cube of any odd number is even. (ii) There is no perfect cube that ends with 8. (iii) The cube of a 2-digit number may be a 3-digit number. (iv) The cube of a 2-digit number may have seven or more digits. (v) Cube numbers have an odd number of factors.

> ***Ans.*** (i) **False**: odd × odd × odd is odd, e.g. 3^3 = 27.
>
> (ii) **False**: 2^3 = 8, 12^3 = 1728 (numbers ending in 2 have cubes ending in 8).
>
> (iii) **False**: the smallest 2-digit number gives 10^3 = 1000, already 4 digits.
>
> (iv) **False**: the largest gives 99^3 = 970299, only 6 digits (7 digits start at 100^3).
>
> (v) **False**: 8 has factors 1, 2, 4, 8 (four). Only cubes that are also squares (1, 64, 729, …) have an odd number of factors.

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4. 1331 is a perfect cube. Guess its cube root without factorisation. Similarly, guess the cube roots of 4913, 12167 and 32768.

> **Ans.** **Method**: (1) the **last digit** of the cube fixes the last digit of the root (table); (2) strike off the last three digits; the number left lies between two cubes and gives the **tens digit**.

Cube	Last digit → root ends in	Left part	Between	Cube root
1331	1 → 1	1	$1^3 \le 1 < 2^3$	**11**
4913	3 → 7	4	$1^3 < 4 < 2^3$	**17**
12167	7 → 3	12	$2^3 < 12 < 3^3$	**23**
32768	8 → 2	32	$3^3 < 32 < 4^3$	**32**

5. Which is the greatest? (i) $67^3 - 66^3$ (ii) $43^3 - 42^3$ (iii) $67^2 - 66^2$ (iv) $43^2 - 42^2$

> **Ans.** Consecutive squares differ by $n + (n + 1)$: $67^2 - 66^2 = 133$, $43^2 - 42^2 = 85$. Consecutive cubes differ much more: $(n + 1)^3 - n^3 = 3n(n + 1) + 1$ (level 1 of the cube differences), so $67^3 - 66^3 = 3 \times 66 \times 67 + 1 = 13267$ and $43^3 - 42^3 = 3 \times 42 \times 43 + 1 = 5419$. Greatest: **(i) $67^3 - 66^3$**.

Puzzle: Square Pairs

> **Ans.** **1 to 17 in a row**, every two neighbours adding to a square: **16, 9, 7, 2, 14, 11, 5, 4, 12, 13, 3, 6, 10, 15, 1, 8, 17** (sums 25, 16, 9, 16, 25, 16, 9, 16, 25, 16, 9, 16, 25, 16, 9, 25). **Only one way** (apart from reversing): 16 can only sit next to 9 ($16 + 9 = 25$) and 17 only next to 8 ($17 + 8 = 25$), so they must be the two ends, and then every step is forced.
>
> **1 to 32 in a circle**: 1, 8, 28, 21, 4, 32, 17, 19, 30, 6, 3, 13, 12, 24, 25, 11, 5, 31, 18, 7, 29, 20, 16, 9, 27, 22, 14, 2, 23, 26, 10, 15 (and $15 + 1 = 16$ closes the circle).

Exam-style questions *Exam-style*

1. What is the units digit of 1237^2? *[1]*

> **Ans.** $7^2 = 49$, so **9**.

2. How many numbers lie between 25^2 and 26^2? *[1]*

> **Ans.** $2 \times 25 =$ **50**.

3. Without finding the square, say whether 1000000 and 100000 can be squares. *[2]*

> **Ans.** 1000000 has 6 zeros (even): it can be, and is: 1000^2. 100000 has **5 zeros** (odd): **not** a square.

4. Find the smallest number by which 8640 must be divided to get a perfect cube. Find the cube root of the quotient. *[3]*

> **Ans.** $8640 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times$ **5**. Divide by **5**: 1728, $\sqrt[3]{1728} = 2 \times 2 \times 3 =$ **12**.

5. Find the smallest number by which 392 must be multiplied to get a perfect cube. *[2]*

> **Ans.** $392 = (2 \times 2 \times 2) \times 7 \times 7$: multiply by **7**; $2744 = 14^3$.

6. A cube-shaped box has volume 3375 cm^3. Find its edge. *[2]*

> **Ans.** $3375 = 3^3 \times 5^3$, edge $= 3 \times 5 =$ **15 cm**.

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7. Find the least square number divisible by 6, 9 and 15. [3]

> **Ans.** LCM = 90 = 2 × (3 × 3) × 5. Unpaired: 2 and 5, so multiply by 2 × 5 = 10: **900** = 30^2.

8. Estimate $\sqrt{500}$ to the nearest whole number. [2]

> **Ans.** $22^2 = 484 < 500 < 529 = 23^2$; 500 is nearer 484 (16 away) than 529 (29 away), so $\sqrt{500} \approx$ **22**.

9. Show with a figure that $1 + 3 + 5 + 7 + 9 = 5^2$, and use the pattern to find the sum of the first 50 odd numbers. Is $1 + 3 + 5 + \ldots + 99$ the same sum? [5]

> **Ans.** Draw a 5 × 5 square and colour it in "L" layers of 1, 3, 5, 7, 9 small squares (figure on page 5): together they fill 5 × 5 = 25. Each new L adds $2n + 1$ squares to an $n \times n$ square, making $(n + 1)^2$. So the first 50 odd numbers add to $50^2 = 2500$. The 50th odd number is $2 \times 50 - 1 = 99$, so **yes**: $1 + 3 + \ldots + 99 = 2500$.

Case study 1: the school courtyard Exam-style. A square courtyard is paved with **1764** square tiles, each 25 cm × 25 cm.

(i) How many tiles are along one side? 1764 = (2 × 2) × (3 × 3) × (7 × 7), $\sqrt{1764} = 2 \times 3 \times 7 =$ **42**. [1]

(ii) Length of one side in metres? 42 × 25 cm = 1050 cm = **10.5 m**. [1]

(iii) How many tiles touch the outer edge? 4 × 42 − 4 = **164** (corners counted once). [1]

(iv) To make the courtyard one tile longer on each side (43 × 43), how many extra tiles? $43^2 - 42^2 = 42 + 43 =$ **85**. [2]

Case study 2: building a cube Exam-style. Meera has many identical bricks of size 2 cm × 3 cm × 4 cm and wants to stack them into a solid cube.

(i) Smallest possible edge of the cube? It must be a multiple of 2, 3 and 4: LCM = **12 cm**. [1]

(ii) How many bricks? $12^3 \div (2 \times 3 \times 4) = 1728 \div 24 =$ **72**. [2]

(iii) Could she use exactly 100 bricks? 100 × 24 = 2400 cm^3, and 2400 is not a cube ($13^3 = 2197$, $14^3 = 2744$): **no**. [2]

Practice set: test yourself *Exam-style*

Try every question before looking at the answers at the end of the chapter. Tick the ones you got right: 40 or more out of 48 means you are ready for the test.

A. Fill in the blanks (1 mark each)

1. The square of an odd number is always _____.
2. There are _____ numbers between 12^2 and 13^2.
3. If a number ends in 3, its cube ends in _____.
4. $\sqrt[3]{(8 \times 27)}$ = _____.
5. The 20th odd number is _____.
6. $(-7)^2$ = _____ and $(-7)^3$ = _____.
7. If $n^2 = 2025$, then n = _____.
8. The smallest taxicab number is _____.

B. True or false? (1 mark each)

1. Every perfect square ends in 0, 1, 4, 5, 6 or 9.

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2. A number ending in 5 zeros can be a perfect square.
3. The sum of two perfect squares is always a perfect square.
4. The product of two perfect squares is always a perfect square.
5. Every perfect cube ends in an odd digit.
6. 1000 has an odd number of factors.
7. The cube of a negative number is negative.
8. There are $2n$ numbers between n^2 and $(n+1)^2$.

C. Very short answer (1 mark each)

1. Units digit of 2468^2?
2. Can 1690 be a perfect square? Why?
3. $\sqrt{0.81} = ?$
4. How many zeros does 4000^2 end with?
5. $1 + 3 + 5 + \ldots + 15 = ?$
6. $\sqrt[3]{(-1000)} = ?$
7. The smallest 3-digit perfect cube?
8. The largest 3-digit perfect square?
9. $(1.2)^3 = ?$
10. Which of 216, 729, 343 is both a square and a cube?

D. Short answer I (2 marks each)

1. Find $\sqrt{5929}$ by prime factorisation.
2. Find the smallest number by which 252 must be multiplied to get a perfect square. Find the square root.
3. Find $\sqrt[3]{(-2744)}$.
4. Find 51^2 using $(50+1)^2 = 50^2 + 2 \times 50 \times 1 + 1^2$.
5. Show that 1331 is a perfect cube and find its cube root.
6. Is 1728 a perfect cube? Find $\sqrt[3]{1728}$.
7. Between which two whole numbers does $\sqrt{90}$ lie? Which one is it nearer?
8. How many perfect squares lie between 200 and 300? Name them.

E. Short answer II (3 marks each)

1. Find the smallest number by which 1620 must be divided so that the quotient is a perfect square. Find the square root of the quotient.
2. Find the smallest perfect square that is divisible by each of 8, 15 and 20.
3. Find the smallest number by which 3087 must be multiplied to make it a perfect cube. Find the cube root of the product.
4. A square field has an area of 5184 m². Find its side and its perimeter.
5. Find (i) $\sqrt[3]{0.027}$ (ii) $\sqrt[3]{(125/343)}$ (iii) $\sqrt{(0.0144)}$.
6. The squares of two consecutive numbers differ by 41. Find the numbers.

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F. Long answer (5 marks each)

1. A gardener has 1000 saplings. He wants to plant them in a square, with as many rows as columns. (i) How many rows can he make, and how many saplings are left over? (ii) How many more saplings would he need to add one more row and one more column?
2. (i) Find the greatest 4-digit perfect square. (ii) Find the least number that must be subtracted from 9999 to get a perfect square. (iii) Find the least number that must be added to 9999 to get a perfect square.
3. A cube-shaped water tank holds 9261 litres when full (1 litre = 1000 cm^3). Find (i) its volume in cm^3 in exponent form (ii) the length of its edge in cm (iii) the area of its base in m^2.
4. Using a figure, show that $(n + 1)^2 - n^2 = 2n + 1$. Use it to find (i) 101^2 from 100^2 (ii) 1001^2 from 1000^2 (iii) the number that must be added to 35^2 to get 36^2.

G. Challenge (HOTS) questions

1. A perfect square has 7 digits. How many digits can its square root have? Explain.
2. Find the smallest number which becomes a perfect square when multiplied by 2 and a perfect cube when multiplied by 3.
3. Find all numbers between 100 and 1000 that are both perfect squares and perfect cubes.
4. In the locker puzzle with 200 lockers and 200 people, how many lockers stay open? Which is the last open locker?

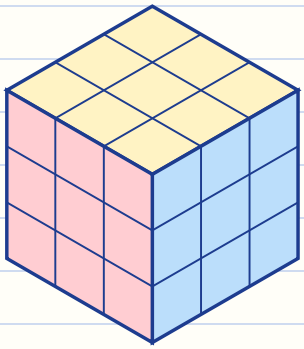
Case study 3: the painted cube Exam-style. A big cube is made by joining 27 small identical cubes (3 × 3 × 3). The whole outside of the big cube is painted red.

(i) How many small cubes have **three** red faces? [1]

(ii) How many small cubes have **no** red face? [1]

(iii) For a 4 × 4 × 4 cube made the same way, how many small cubes are there, and how many have no red face? [2]

(iv) A painted n × n × n cube has 125 small cubes with no red face. Find n and the total number of small cubes. [2]



3 × 3 × 3 = 27

the 3 × 3 × 3 cube of Case study 3: corners have 3 painted faces

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20 MCQs

1. Which can be a perfect square?
   (a) 3257 (b) 6728 (c) 4489 (d) 5332

2. A square never ends in
   (a) 7 (b) 6 (c) 1 (d) 9

3. The number of zeros at the end of a perfect square is always
   (a) odd (b) 3 (c) 1 (d) even

4. $1 + 3 + 5 + \ldots + 19 =$
   (a) 81 (b) 100 (c) 121 (d) 361

5. How many numbers lie between 10^2 and 11^2?
   (a) 10 (b) 20 (c) 11 (d) 21

6. The smallest number to multiply 180 by to get a square is
   (a) 2 (b) 3 (c) 10 (d) 5

7. $\sqrt{1764} =$
   (a) 42 (b) 38 (c) 44 (d) 48

8. The cube of a number ending in 7 ends in
   (a) 7 (b) 9 (c) 3 (d) 1

9. $\sqrt[3]{2197} =$
   (a) 11 (b) 19 (c) 17 (d) 13

10. The Hardy-Ramanujan number is
    (a) 1728 (b) 1729 (c) 4104 (d) 1331

11. Which is not a perfect cube?
    (a) 216 (b) 1000 (c) 500 (d) 3375

12. $(-5)^3 =$
    (a) −125 (b) 125 (c) −15 (d) 15

13. The smallest number to multiply 72 by to get a perfect cube is
    (a) 2 (b) 3 (c) 6 (d) 9

14. $\sqrt{0.0064} =$
    (a) 0.8 (b) 0.008 (c) 0.08 (d) 8

15. How many perfect squares are there from 1 to 100?
    (a) 9 (b) 50 (c) 11 (d) 10

16. Which is a perfect cube?
    (a) 27000 (b) 9000 (c) 2700 (d) 900

17. $99^2 =$
    (a) 9801 (b) 9081 (c) 9901 (d) 9810

18. A perfect cube ends in 6. Its cube root ends in
    (a) 4 (b) 2 (c) 8 (d) 6

19. For cubes, successive differences become all equal at level
    (a) 1 (b) 3 (c) 2 (d) 4

20. $\sqrt{1296} =$
    (a) 34 (b) 44 (c) 36 (d) 46

1 (c) · 2 (a) · 3 (d) · 4 (b) · 5 (b) · 6 (d) · 7 (a) · 8 (c) · 9 (d) · 10 (b) · 11 (c) · 12 (a) · 13 (b) · 14 (c) · 15 (d) · 16 (a) · 17 (a) · 18 (d) · 19 (b) · 20 (c)

Assertion–Reason (6)

(a) Both true, R explains A. (b) Both true, R does not explain A. (c) A true, R false. (d) A false, R true.

1. A: 2058 is not a perfect square. R: A perfect square never ends in 8.
2. A: 36 has an odd number of factors. R: Every factor of a number has a partner factor.
3. A: 1000 is a perfect cube. R: A perfect cube can end in exactly two zeros.
4. A: 64 is both a square and a cube. R: $64 = 8^2 = 4^3$.
5. A: 3 is the smallest number by which 9408 must be multiplied to get a perfect square. R: In $9408 = 2^6 \times 3 \times 7^2$, only the prime 3 has no partner.
6. A: $81 + 144 = 225$ is a perfect square. R: The sum of two perfect squares is always a perfect square.

*Key: 1 (a) · 2 (b): R is true, but A is true because 6 is its **own** partner, which R does not explain · 3 (c) · 4 (a) · 5 (a) · 6 (c): $225 = 15^2$, but $1 + 4 = 5$ is not a square*

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Answers to the practice set

A. 1. odd 2. 24 3. 7 4. 6 5. 39 6. 49; −343 7. 45 8. 1729

B. 1. T 2. F (odd number of zeros) 3. F (1 + 4 = 5) 4. T ($a^2b^2 = (ab)^2$) 5. F (8, 64) 6. F (1000 = $2^3 \times 5^3$ has 4 × 4 = 16 factors; it is not a square) 7. T 8. T

C. 1. 4 ($8^2 = 64$) 2. No: it ends in one zero (odd) 3. 0.9 4. Six (4000^2 = 1,60,00,000) 5. 64 (eight odd numbers: 8^2) 6. −10 7. 125 8. 961 (31^2) 9. 1.728 10. $729 = 27^2 = 9^3$

D. 1. $5929 = 7 \times 7 \times 11 \times 11$, $\sqrt{} = 7 \times 11 = 77$

2. $252 = 2^2 \times 3^2 \times 7$: multiply by 7; $\sqrt{1764} = 42$

3. $2744 = 2^3 \times 7^3$: $\sqrt[3]{(-2744)} = -14$

4. 2500 + 100 + 1 = 2601

5. $1331 = 11 \times 11 \times 11$, $\sqrt[3]{} = 11$

6. $1728 = 2^6 \times 3^3 = (2^2 \times 3)^3$: yes, 12

7. 81 < 90 < 100: between 9 and 10, nearer 9 (9 away from 81, 10 away from 100)

8. Three: 225, 256, 289

E. 1. $1620 = 2^2 \times 3^4 \times 5$: divide by 5; $324 = 18^2$

2. LCM = $120 = 2^3 \times 3 \times 5$; unpaired 2, 3, 5: $120 \times 30 = 3600 = 60^2$

3. $3087 = 3^2 \times 7^3$: multiply by 3; 9261, $\sqrt[3]{} = 21$

4. $5184 = 2^6 \times 3^4$, side $2^3 \times 3^2 = 72$ m, perimeter 288 m

5. (i) 0.3 (ii) 5/7 (iii) 0.12

6. n + (n + 1) = 41: 20 and 21

F. 1. $31^2 = 961 \le 1000 < 1024 = 32^2$: (i) 31 rows, 39 left over (ii) $32^2 - 1000 = 24$ more

2. (i) $99^2 = 9801$ (ii) 9999 − 9801 = 198 (iii) $100^2 - 9999 = 1$

3. (i) $9261 \times 1000 = 9261000$ cm^3 = $21^3 \times 10^3 = 210^3$ (ii) 210 cm (iii) 210×210 cm^2 = 44100 cm^2 = 4.41 m^2

4. L-figure (page 5). (i) 10000 + 201 = 10201 (ii) 1000000 + 2001 = 1002001 (iii) 2 × 35 + 1 = 71

G. 1. 4 digits: $1000^2 = 10^6$ is the smallest 7-digit square and 3162^2 = 99,98,244 is the largest (3163^2 has 8 digits)

2. $n = 2^a \times 3^b$ with a + 1 even, a a multiple of 3 ⇒ a = 3; b even, b + 1 a multiple of 3 ⇒ b = 2: $n = 8 \times 9 =$ **72** ($144 = 12^2$, $216 = 6^3$)

3. Only **729** ($= 27^2 = 9^3$); 64 is below 100, 4096 above 1000

4. Squares up to 200: 1^2, …, $14^2 = 196$, so **14 lockers**; the last is 196

Case study 3. (i) 8 (the corners) (ii) 1 (the centre) (iii) 64 small cubes; inner 2 × 2 × 2 = 8 unpainted (iv) $(n-2)^3 = 125 \Rightarrow n - 2 = 5$, $n = 7$; total $7^3 = 343$

Formula sheet and last-night summary

- Square: $n^2 = n \times n$. Perfect squares end in 0, 1, 4, 5, 6, 9 and in an even number of zeros. Only squares have an odd number of factors.
- 1 + 3 + 5 + … (n odd numbers) = n^2; the n-th odd number is 2n − 1; $(n+1)^2 = n^2 + 2n + 1$; 2n numbers lie between n^2 and $(n+1)^2$.
- Consecutive triangular numbers add to a square. $(n5)^2 = n(n+1)$ followed by 25.
- $\sqrt{}$: perfect square ⇔ prime factors pair up; root = one factor from each pair. Estimate with $n^2 < N < (n+1)^2$.
- Cube: $n^3 = n \times n \times n$; last digits 2 ↔ 8, 3 ↔ 7, others stay. A cube ends in 0, 3, 6, … zeros. $(-n)^3 = -n^3$.
- $\sqrt[3]{}$: perfect cube ⇔ prime factors form triplets; root = one factor from each triplet. Guess $\sqrt[3]{}$ of a 4–6 digit cube from its last digit and the part left after striking off three digits.

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- Smallest multiplier/divisor: the product of the leftover (unpaired / un-tripled) primes, each raised to what completes the group.
- $1729 = 1^3 + 12^3 = 9^3 + 10^3$ (smallest taxicab number). Differences of squares settle at level 2 (all 2), of cubes at level 3 (all 6).

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