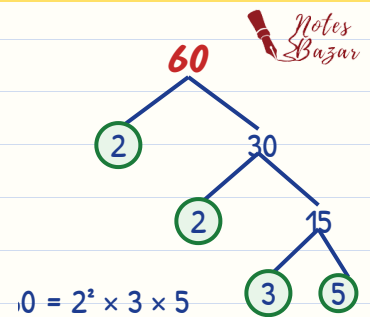


Class 10 · Mathematics · Number Systems

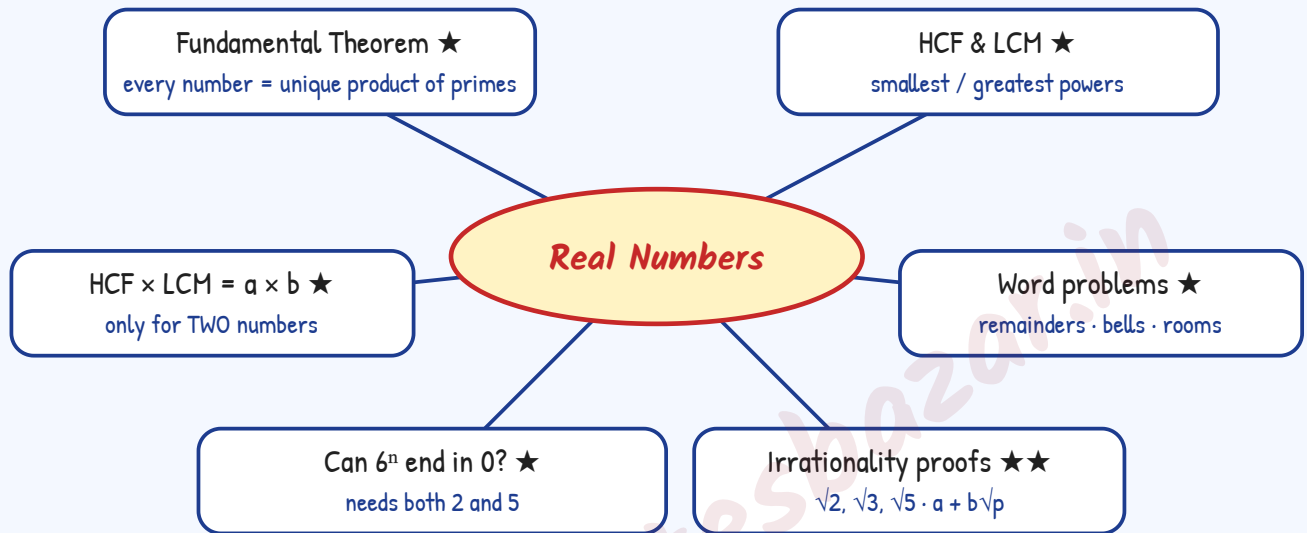
Chapter 1

Real Numbers

NCERT Mathematics, Reprint 2026-27 · CBSE board 2027



Chapter map (★ = asked again and again)



What is inside

1. Concept notes with worked examples (p. 2-7)
2. Don't-confuse table & formula box (p. 7)
3. NCERT Exercise 1.1 and 1.2 + HOTS, solved (p. 8-9)
4. Board questions 2023-2025, solved (p. 10-12)
5. Real board case study + 2 exam-style cases (p. 13)
6. 20 MCQs · 10 Assertion-Reason · fill-ups (p. 14-15)
7. 1-, 2-, 3- and 5-mark practice, with answers (p. 15-16)
8. My mistakes page · last-night summary (p. 17)

Colour code used

red = key term green = definition

purple = sub-heading black = working

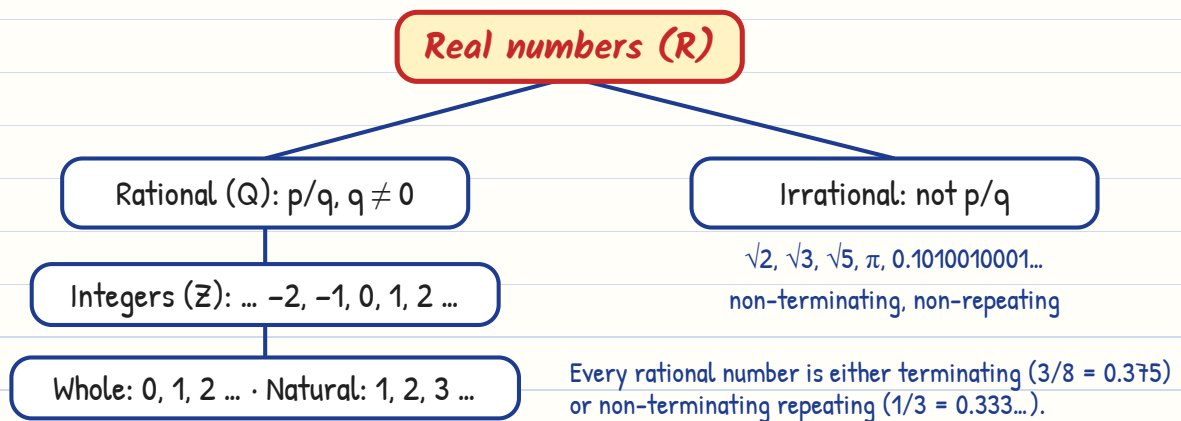
★ = high-frequency board topic

! = mark the examiner gives for that step

★ **Exam tip:** I checked 17 real CBSE Standard papers of 2023, 2024 and 2025. **Every single one** had questions from this chapter (2 to 6 marks), and an irrationality proof (2 or 3 marks) came in 15 of them. Learn the proof template on p. 6 word for word.

Board questions on p. 10-13 are taken from the actual CBSE Mathematics (Standard, 041) question papers and marking schemes of 2023, 2024 and 2025; the paper code is shown with each.

1.1 The family of numbers



Definition ✎ **Prime number:** a natural number greater than 1 with exactly two factors, 1 and itself. 2, 3, 5, 7, 11, 13, ... (2 is the only even prime).

Definition ✎ **Composite number:** a natural number greater than 1 with more than two factors. 4, 6, 8, 9, 10, ... (4 is the smallest composite).

Definition ✎ **Co-prime numbers:** two numbers whose only common factor is 1, i.e. HCF = 1. e.g. 8 and 15 (neither need be prime).

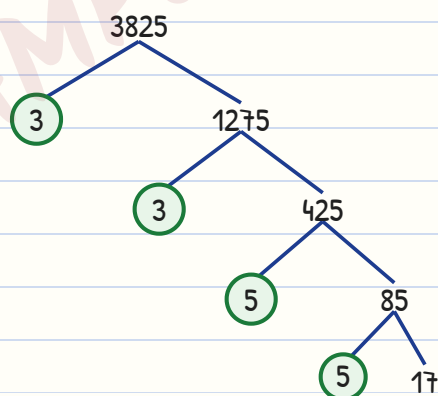
1 is neither prime nor composite. So "the least prime is 2", "the least composite is 4", and the HCF of two co-primes is **1**, not 0.

1.2 The Fundamental Theorem of Arithmetic ★

Definition ✎ **Theorem 1.1:** Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

"Unique" is the power of the theorem: **60** can be written as $2 \times 2 \times 3 \times 5$ or $5 \times 3 \times 2 \times 2$, but never with a 7 in it, and never with three 2s.

How to factorise: the factor tree / repeated division



Factor tree: $3825 = 3^2 \times 5^2 \times 17$ (NCERT Ex 1.1 Q1)

Divide by	3825
3	1275
3	425
5	85
5	17
17	1

★ **Exam tip:** Always divide by the **smallest** prime first (2, 3, 5, 7, 11, 13, 17, 19, 23 ...). Write the answer in **index form**: $3^2 \times 5^2 \times 17$. Examiners give the mark only for the final product.

1.3 HCF and LCM by prime factorisation ★

Definition ✎ **HCF** = product of the smallest power of each common prime factor.

Definition ✎ **LCM** = product of the greatest power of every prime factor that appears.

Worked example: HCF and LCM of 96 and 120 CBSE 2023 · 30/5/1

$$96 = 2^5 \times 3 \quad \text{and} \quad 120 = 2^3 \times 3 \times 5 \quad \text{①}$$

Prime	in 96	in 120	HCF takes (smaller)	LCM takes (greater)
2	2^5	2^3	2^3	2^5
3	3^1	3^1	3^1	3^1
5	—	5^1	— (not common)	5^1

$$\text{HCF} = 2^3 \times 3 = 24, \quad \text{LCM} = 2^5 \times 3 \times 5 = 480 \quad \text{①}$$

The product rule (two numbers only!) ★

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b \quad \text{check: } 24 \times 480 = 11520 = 96 \times 120 \checkmark$$

X Don't do this: Never use this rule for three numbers. For 12, 15, 21: HCF = 3, LCM = 420, HCF $\times$ LCM = 1260, but $12 \times 15 \times 21 = 3780$. The board asked exactly this trap in 2025 (p. 13).

Using the rule to find a missing value

NCERT Ex 1.1 Q4 HCF (306, 657) = 9. Find LCM.

$$\text{LCM} = \frac{306 \times 657}{9} = 34 \times 657 = 22338$$

CBSE 2024 · 30/4/1 HCF (65, 104) = 13, LCM = 40x. Find x.

$$\text{LCM} = \frac{65 \times 104}{13} = 520 = 40x \Rightarrow x = 13$$

Three quick facts that save time in MCQs

Situation	HCF	LCM	Example
a, b co-prime	1	$a \times b$	8, 15 $\rightarrow$ 1, 120
a is a factor of b	a	b	12, 36 $\rightarrow$ 12, 36
two different primes p, q	1	pq	7, 11 $\rightarrow$ 1, 77
numbers $a = 2^m 3^n$ given in index form	just compare powers: smallest for HCF, greatest for LCM		

Algebraic version CBSE 2025 · 30/4/1

$p = x^2 y^3, q = x y^4, r = x^5 y^2$ (x, y primes): HCF = $x y^2$ (smallest powers), LCM = $x^5 y^4$ (greatest powers).

HCF $\times$ LCM = $x^6 y^6$ but $p \times q \times r = x^8 y^9$, so HCF $\times$ LCM $\neq$ pqr: the rule fails for three numbers.

1.4 Word problems: HCF or LCM? ★

Key words in the question	Use	Why
greatest / largest number that divides ...; maximum size of each group, row, room, piece; minimum number of rooms, rows, groups	HCF	the answer must go <u>into</u> all the numbers
smallest / least number divisible by ...; when will bells ring / lights flash / runners meet together again ; minimum length that can be measured exactly	LCM	the answer must be a <u>multiple</u> of all the numbers

Type 1: "divides ... leaving remainders" ★

If a number leaves remainders r_1 and r_2 when it divides a and b , then it divides $a - r_1$ and $b - r_2$ exactly. So the answer is **HCF** ($a - r_1$, $b - r_2$).

CBSE 2023 · 30/1/1 Greatest number that divides 85 and 72 leaving remainders 1 and 2. **CBSE 2024 · 30/5/1** Greatest number that divides 281 and 1249, leaving remainders 5 and 7.

$$\begin{aligned} \text{HCF}(84, 70): 84 &= 2^2 \times 3 \times 7, 70 = 2 \times 5 \times 7 \quad \text{①} & \text{HCF}(276, 1242): 276 &= 2^2 \cdot 3 \cdot 23, 1242 = 2 \cdot 3^3 \cdot 23 \\ \text{HCF} &= 2 \times 7 = \mathbf{14} \quad \text{①} & \text{HCF} &= 2 \times 3 \times 23 = \mathbf{138} \end{aligned}$$

Type 2: "together again" (LCM)

NCERT Ex 1.1 Q7 Sonia takes 18 min and Ravi 12 min to drive one round of a field. They start together. When do they meet again at the start?

$$18 = 2 \times 3^2, 12 = 2^2 \times 3 \Rightarrow \text{LCM} = 2^2 \times 3^2 = 36. \text{ They meet after } \mathbf{36} \text{ minutes.}$$

Type 3: "minimum number of rooms" (HCF, then divide) **CBSE 2024 · 30/1/1**

In a seminar, 48, 80 and 144 participants (of three different subjects) are to be seated in rooms; each room has the same number of participants, all of one subject. Find the minimum number of rooms.

$$\begin{aligned} \text{Most people per room} &= \text{HCF}(48, 80, 144): 48 = 2^4 \cdot 3, 80 = 2^4 \cdot 5, 144 = 2^4 \cdot 3^2 \Rightarrow \text{HCF} = 2^4 = 16 \quad \text{②} \\ \text{Rooms} &= \frac{48}{16} + \frac{80}{16} + \frac{144}{16} = 3 + 5 + 9 = \mathbf{17} \quad \text{①} \end{aligned}$$

Type 4: ratio given

CBSE 2023 · 30/4/1 Two numbers are in the ratio 2 : 3 and their LCM is 180. Find their HCF. **CBSE 2025 · 30/3/1** Two numbers are in the ratio 4 : 5 and their HCF is 11. Find their LCM.

$$\begin{aligned} \text{Numbers } 2x, 3x \text{ (HCF} = x\text{), LCM} &= 6x = 180 \Rightarrow \text{Numbers } 44 \text{ and } 55 \Rightarrow \text{LCM} = 4 \times 5 \times 11 = \mathbf{220}. \\ x &= 30. \text{ Numbers } 60, 90; \text{HCF} = \mathbf{30}. \end{aligned}$$

★ **Exam tip:** In ratio questions, write the numbers as ax and bx with a, b co-prime. Then $\text{HCF} = x$ and $\text{LCM} = abx$.

1.5 Can a power end with the digit 0? ★

A number ends with 0 $\Leftrightarrow$ it is divisible by $10 = 2 \times 5 \Leftrightarrow$ its prime factorisation contains **both 2 and 5**.

Full-marks template CBSE 2023 · 30/6/11 CBSE 2024 · 30/2/11

Check whether 6^n can end with the digit 0 for any natural number n . [2]

1. If 6^n ended with 0, it would be divisible by 10, i.e. its prime factorisation would contain the prime 5. ½
2. But $6^n = (2 \times 3)^n = 2^n \times 3^n$: its only prime factors are 2 and 3. ½
3. By the **Fundamental Theorem of Arithmetic**, this factorisation is unique, so 5 can never appear. ½
4. Hence there is no natural number n for which 6^n ends with 0. ½

Number	Prime factors	Can it end in 0?	Missing prime
4^n	2 only	No	5
$6^n, 12^n$	2, 3	No	5
15^n CBSE 2024 · 30/2/11	3, 5	No	2
$5^n, 25^n$	5 only	No	2
$10^n, 20^n$	2 and 5	Yes (always)	—

✗ Don't do this: Writing " 6^n ends in 6 because $6 \times 6 = 36$ " gets 0 marks: the question is about prime factorisation. Always quote the Fundamental Theorem of Arithmetic.

1.6 Showing a number is composite

Take out the common factor. If the number is a product of two numbers other than 1 and itself, it is composite.

NCERT Ex 1.1 Q6 Why is $7 \times 11 \times 13 + 13$ composite?

$7 \times 11 \times 13 + 13 = 13(77 + 1) = 13 \times 78 = 2 \times 3 \times 13^2$: it has factors other than 1 and itself, so it is **composite**.

Similarly $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5(1008 + 1) = 5 \times 1009$: composite.

Sum of exponents CBSE 2025 · 30/2/11

$4004 = 2^2 \times 7 \times 11 \times 13 \Rightarrow$ sum of exponents $= 2 + 1 + 1 + 1 = 5$.

★ **Exam tip:** Before declaring a number prime, test division by every prime up to its square root. For 1009: $\sqrt{1009} \approx 31.8$, and none of 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 divides it, so 1009 is prime.

1.7 Proving a number is irrational ★★

Definition **Theorem 1.2:** Let p be a prime. If p divides a^2 , then p divides a (for any positive integer a).

Why: write $a = p_1 p_2 \cdots p_n$ (primes). Then $a^2 = p_1 p_1 p_2 p_2 \cdots p_n p_n$. If p divides a^2 , then by uniqueness p is one of $p_1, \dots, p_n$, so p divides a .

Template 1: prove $\sqrt{5}$ is irrational (3 marks) CBSE 2023 · 30/1/1 2023 · 30/4/1 2024 · 30/2/1

2025 · 30/6/1

1. Assume, on the contrary, that $\sqrt{5}$ is rational. Then $\sqrt{5} = \frac{p}{q}$, where p, q are integers, $q \neq 0$, and p, q are **co-prime**. $\frac{1}{2}$
2. Squaring: $5 = \frac{p^2}{q^2} \Rightarrow p^2 = 5q^2 \dots$ (i). So 5 divides p^2 , hence 5 divides p (Theorem 1.2). $\frac{1}{1}$
3. Let $p = 5c$. Then $25c^2 = 5q^2 \Rightarrow q^2 = 5c^2$, so 5 divides q^2 , hence 5 divides $q \dots$ (ii). $\frac{1}{1}$
4. From (i) and (ii), 5 is a common factor of p and q . This contradicts the fact that p and q are co-prime. So our assumption is wrong: $\sqrt{5}$ is **irrational**. $\frac{1}{2}$

Same steps for $\sqrt{2}$ 2023 · 30/6/1 2025 · 30/4/1 and $\sqrt{3}$ 2023 · 30/5/1 2024 · 30/5/1 2025 · 30/2/1 2025 · 30/5/1 : just change 5 to 2 or 3.

Template 2: $a + b\sqrt{p}$ type, using a known irrational CBSE 2023 · 30/2/1 2024 · 30/1/1

Prove that $5 - 2\sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational.

1. Assume $5 - 2\sqrt{3}$ is rational: $5 - 2\sqrt{3} = \frac{a}{b}$, with a, b integers, $b \neq 0$. $\frac{1}{2}$
2. Then $\sqrt{3} = \frac{5b - a}{2b}$. $\frac{1}{1}$
3. The right-hand side is rational (integers $5b - a$ and $2b \neq 0$), but $\sqrt{3}$ is irrational: a contradiction. $\frac{1}{1}$
4. Hence $5 - 2\sqrt{3}$ is irrational. $\frac{1}{2}$

Board variants (same template)

$$\begin{aligned} 2 + \sqrt{3} & \text{ 2023 · 30/2/1} \\ \frac{2 - \sqrt{3}}{5} & \text{ 2024 · 30/4/1 : } \sqrt{3} = 2 - \frac{5a}{b} \\ 5\sqrt{3} + \frac{2}{3} & \text{ 2025 · 30/3/1 : } \sqrt{3} = \frac{3a - 2b}{15b} \\ (\sqrt{2} + \sqrt{3})^2 = 5 + 2\sqrt{6} & \text{ 2024 · 30/5/1 : } \sqrt{6} = \frac{a - 5b}{2b} \end{aligned}$$

$\frac{1}{\sqrt{5}}$ is irrational 2025 · 30/1/1

Assume $\frac{1}{\sqrt{5}} = \frac{a}{b}$ (rational, $a \neq 0$). Then $\sqrt{5} = \frac{b}{a}$, which is rational: contradiction with $\sqrt{5}$ irrational. (NCERT 1.2 Q3(i))

uses $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ the same way.)

Proof checklist: tick every line before you move on

Template 1 ($\sqrt{p}$)	Template 2 ($a + b\sqrt{p}$)
☐ "Assume, on the contrary ..."	☐ "Assume ... is rational = $\frac{a}{b}$, $b \neq 0$ "
☐ p, q co-prime, $q \neq 0$	☐ isolate $\sqrt{p}$ on one side
☐ $p^2 = 5q^2 \Rightarrow 5$ divides p (quote Theorem 1.2)	☐ "RHS is rational as a, b are integers"
☐ put $p = 5c \Rightarrow 5$ divides q	☐ "but $\sqrt{p}$ is irrational (given / proved)"

Both templates end with: ☐ "... a contradiction. Hence ... is irrational."

X Don't do this: Never skip "p, q co-prime": without it there is no contradiction (−1½ marks).

Don't-confuse table

Looks similar	But
Prime vs co-prime	A prime has exactly 2 factors; co-primes are a pair with HCF 1 (8 and 9 are co-prime, neither is prime).
HCF vs LCM powers	HCF: smallest power of common primes. LCM: greatest power of all primes.
HCF $\times$ LCM = product	True for two numbers only.
Rational $\times$ irrational	Non-zero rational $\times$ irrational = irrational ($7\sqrt{5}$). But irrational $\times$ irrational can be rational: $\sqrt{2} \times \sqrt{8} = 4$, $(2 + \sqrt{3})(2 - \sqrt{3}) = 1$.
Rational + irrational	Always irrational ($6 + \sqrt{2}$). But irrational + irrational can be rational: $\sqrt{2} + (-\sqrt{2}) = 0$.
π vs $\frac{22}{7}$	π is irrational; $\frac{22}{7}$ is only a rational approximation of it.
$\sqrt{4}$, $\sqrt{9}$, $\sqrt{0.25}$	All rational (2, 3, 0.5). Only square roots of non-perfect squares like $\sqrt{2}$, $\sqrt{8}$, $\sqrt{12}$ are irrational.
"least prime" vs "least composite"	2 and 4 (1 is neither).

★ Formula box (copy on your revision card)

- $n = p_1^a p_2^b p_3^c \dots$ (unique, FTA)
- HCF = $\prod$ (smallest powers of common primes)
- LCM = $\prod$ (greatest powers of all primes)
- $\text{HCF}(a, b) \times \text{LCM}(a, b) = ab$
- Divides a, b leaving $r_1, r_2 \Rightarrow \text{HCF}(a - r_1, b - r_2)$
- Ends in 0 $\Leftrightarrow$ prime factors include 2 **and** 5
- p prime, $p \mid a^2 \Rightarrow p \mid a$
- $\sqrt{p}$ irrational for every prime p
- rational $\pm$ irrational = irrational
- non-zero rational $\times$ irrational = irrational

Mental-maths drill: HCF and LCM in 10 seconds

Pair	18, 24	15, 25	16, 40	9, 28	36, 48	21, 35
HCF	6	5	8	1	12	7
LCM	72	75	80	252	144	105

Check yourself (answers upside down at the end of the line)

- Is $\sqrt{2} + \sqrt{2}$ rational? (No: $2\sqrt{2}$)
- Find two irrational numbers whose sum is rational. ($3 + \sqrt{2}$ and $3 - \sqrt{2}$: sum 6)
- If $\text{HCF}(a, b) = a$, what is $\text{LCM}(a, b)$? (b , since a divides b)
- Can the HCF of two numbers be greater than their LCM? (Never: $\text{HCF} \leq \text{smaller number} \leq \text{LCM}$)

★ **Exam tip:** Quick check in the exam: LCM must be a multiple of both numbers, HCF must divide both, and HCF always divides LCM. If your answer fails any of these, recheck the powers.

NCERT Exercise 1.1

Q1. Express each number as a product of its prime factors: (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Ans.

(i) $140 = 2^2 \times 5 \times 7$ (ii) $156 = 2^2 \times 3 \times 13$ (iii) $3825 = 3^2 \times 5^2 \times 17$

(iv) $5005 = 5 \times 7 \times 11 \times 13$ (v) $7429 = 17 \times 19 \times 23$ (try 17: $7429 \div 17 = 437 = 19 \times 23$)

Q2. Find the LCM and HCF of the following pairs and verify that $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$: (i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Ans.

Pair	Factorisation	HCF	LCM	HCF $\times$ LCM = product
26, 91	$2 \cdot 13; 7 \cdot 13$	13	182	$13 \times 182 = 2366 = 26 \times 91 \checkmark$
510, 92	$2 \cdot 3 \cdot 5 \cdot 17; 2^2 \cdot 23$	2	23460	$2 \times 23460 = 46920 = 510 \times 92 \checkmark$
336, 54	$2^4 \cdot 3 \cdot 7; 2 \cdot 3^3$	6	3024	$6 \times 3024 = 18144 = 336 \times 54 \checkmark$

Q3. Find the LCM and HCF by the prime factorisation method: (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Ans.

(i) $12 = 2^2 \cdot 3, 15 = 3 \cdot 5, 21 = 3 \cdot 7 \Rightarrow \text{HCF} = 3, \text{LCM} = 2^2 \cdot 3 \cdot 5 \cdot 7 = 420$

(ii) All three are primes $\Rightarrow \text{HCF} = 1, \text{LCM} = 17 \times 23 \times 29 = 11339$

(iii) $2^3, 3^2, 5^2$: no common prime $\Rightarrow \text{HCF} = 1, \text{LCM} = 8 \times 9 \times 25 = 1800$

Q4. Given that $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$.

Ans. $\text{LCM} = \frac{306 \times 657}{9} = 34 \times 657 = 22338$

Q5. Check whether 6^n can end with the digit 0 for any natural number n .

Ans. $6^n = 2^n \times 3^n$ has no prime factor 5. A number ending in 0 must have both 2 and 5 as prime factors. By the uniqueness of prime factorisation (FTA), 6^n can **never** end with 0. (Full template: p. 5.)

Q6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Ans. $7 \times 11 \times 13 + 13 = 13 \times 78$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times 1009$. Each has a factor other than 1 and itself, so both are **composite**.

Q7. Sonia takes 18 minutes and Ravi 12 minutes to drive one round of a sports field. They start together, at the same point and in the same direction. After how many minutes will they meet again at the starting point?

Ans. They meet at the start after $\text{LCM}(18, 12)$ minutes. $18 = 2 \cdot 3^2, 12 = 2^2 \cdot 3 \Rightarrow \text{LCM} = 2^2 \cdot 3^2 = 36$. They meet after **36 minutes**.

NCERT Exercise 1.2

Q1. Prove that $\sqrt{5}$ is irrational.

Ans.

Suppose $\sqrt{5}$ is rational: $\sqrt{5} = \frac{a}{b}$, with a, b co-prime integers, $b \neq 0$.

Then $5b^2 = a^2$, so 5 divides a^2 , hence 5 divides a (Theorem 1.2). Write $a = 5c$.

Then $5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$, so 5 divides b^2 , hence 5 divides b .

So 5 divides both a and b , which contradicts that they are co-prime. Hence $\sqrt{5}$ is **irrational**.

Q2. Prove that $3 + 2\sqrt{5}$ is irrational.

Ans.

Suppose $3 + 2\sqrt{5} = \frac{a}{b}$ (rational; a, b integers, $b \neq 0$).

Then $2\sqrt{5} = \frac{a}{b} - 3 \Rightarrow \sqrt{5} = \frac{a - 3b}{2b}$.

The right side is rational, so $\sqrt{5}$ would be rational. This contradicts Q1. Hence $3 + 2\sqrt{5}$ is **irrational**.

Q3. Prove that the following are irrationals: (i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Ans.

(i) Suppose $\frac{1}{\sqrt{2}} = \frac{a}{b}$ (a, b integers, $a, b \neq 0$). Then $\sqrt{2} = \frac{b}{a}$, which is rational. But $\sqrt{2}$ is irrational: contradiction. So $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Suppose $7\sqrt{5} = \frac{a}{b}$. Then $\sqrt{5} = \frac{a}{7b}$, which is rational: contradiction. So $7\sqrt{5}$ is irrational.

(iii) Suppose $6 + \sqrt{2} = \frac{a}{b}$. Then $\sqrt{2} = \frac{a - 6b}{b}$, which is rational: contradiction. So $6 + \sqrt{2}$ is irrational.

Theorem 1.3: proving $\sqrt{2}$ is irrational (NCERT example)

Let $\sqrt{2} = \frac{a}{b}$, a, b co-prime. Squaring, $2b^2 = a^2 \Rightarrow 2$ divides $a^2 \Rightarrow 2$ divides a . Let $a = 2c$; then $2b^2 = 4c^2 \Rightarrow b^2 = 2c^2 \Rightarrow 2$ divides b . So 2 is a common factor of a and b : contradiction. Hence $\sqrt{2}$ is irrational.

★ **Exam tip:** For Template 2 questions, the board almost always writes "given that $\sqrt{3}$ is irrational". If it does **not** say so (e.g. NCERT 1.2 Q2), first prove $\sqrt{5}$ is irrational (or quote the proved result) and then use it. Writing the full proof of $\sqrt{5}$ is the safe choice for a 3-mark question.

Why does the contradiction method work?

A statement is either true or false. If assuming it is false (here, "the number is rational") leads to something impossible (two co-prime numbers with a common factor 5), then the assumption must be wrong, so the statement is true. This is called **proof by contradiction**.

HOTS (higher-order) practice

H1. Show that $n^2 - 1$ is divisible by 8 when n is an odd positive integer.

Ans. Let $n = 2k + 1$. Then $n^2 - 1 = 4k^2 + 4k = 4k(k + 1)$. One of $k, k + 1$ is even, so $k(k + 1) = 2m$ and $n^2 - 1 = 8m$.

H2. The HCF of two numbers is 23 and their LCM is 1449. If one number is 161, find the other.

Ans. Other = $\frac{23 \times 1449}{161} = 207$.

Board questions: 1-mark MCQs ★

CBSE 2023 · 30/4/I The ratio of HCF to LCM of the least composite number and the least prime number is: (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

Ans. Least composite 4, least prime 2: HCF 2, LCM 4 $\Rightarrow 2 : 4 =$ (a) 1 : 2

CBSE 2024 · 30/2/I 2024 · 30/3/I If HCF (2520, 6600) = 40 and LCM (2520, 6600) = $252 \times k$, then the value of k is:

Ans. LCM = $\frac{2520 \times 6600}{40} = 415800 = 252 \times 1650 \Rightarrow k = 1650$

CBSE 2024 · 30/3/I If $a = 2^2 \times 3^x$, $b = 2^2 \times 3 \times 5$, $c = 2^2 \times 3 \times 7$ and LCM (a, b, c) = 3780, then x is equal to: (A) 1 (B) 2 (C) 3 (D) 0

Ans. LCM = $2^2 \times 3^x \times 5 \times 7 = 140 \times 3^x = 3780 \Rightarrow 3^x = 27 \Rightarrow$ (C) 3

CBSE 2024 · 30/5/I The greatest number which divides 281 and 1249, leaving remainder 5 and 7 respectively, is: (A) 23 (B) 276 (C) 138 (D) 69

Ans. HCF (276, 1242) = (C) 138 (worked on p. 4)

CBSE 2024 · 30/5/I The LCM of three numbers 28, 44, 132 is: (A) 258 (B) 231 (C) 462 (D) 924

Ans. $2^2 \cdot 7, 2^2 \cdot 11, 2^2 \cdot 3 \cdot 11 \Rightarrow$ LCM = $2^2 \cdot 3 \cdot 7 \cdot 11 =$ (D) 924

CBSE 2024 · 30/5/I If the product of two co-prime numbers is 553, then their HCF is:

Ans. Co-prime $\Rightarrow$ HCF = 1 (the product does not matter)

CBSE 2025 · 30/1/I 2025 · 30/1/I2 If HCF (98, 28) = m and LCM (98, 28) = n , then the value of $n - 7m$ is:

Ans. $98 = 2 \cdot 7^2, 28 = 2^2 \cdot 7 \Rightarrow m = 14, n = 196 \Rightarrow n - 7m = 196 - 98 = 98$

CBSE 2025 · 30/1/I2 The greatest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is: (A) 13 (B) 65 (C) 875 (D) 1750

Ans. HCF (65, 117) = 13 $\Rightarrow$ (A) 13

CBSE 2025 · 30/2/I The sum of the exponents of prime factors in the prime factorisation of 4004 is: (A) 5 (B) 4 (C) 3 (D) 2

Ans. $4004 = 2^2 \cdot 7 \cdot 11 \cdot 13 \Rightarrow$ (A) 5

CBSE 2025 · 30/2/I The HCF of 40, 110 and 360 is:

Ans. $2^3 \cdot 5, 2 \cdot 5 \cdot 11, 2^3 \cdot 3^2 \cdot 5 \Rightarrow$ HCF = $2 \cdot 5 = 10$

CBSE 2025 · 30/3/I If x is the LCM of 4, 6, 8 and y is the LCM of 3, 5, 7 and p is the LCM of x and y , then which is true? (A) $p = 35x$ (B) $p = 4y$ (C) $p = 8x$ (D) $p = 16y$

Ans. $x = 24, y = 105, p = \text{LCM}(24, 105) = 840 = 35 \times 24 \Rightarrow$ (A) $p = 35x$

CBSE 2023 · 30/5/I Assertion (A): The number 5^n cannot end with the digit 0, where n is a natural number. Reason (R): Prime factorisation of 5 has only two factors, 1 and 5.

Ans. (c) A is true, but R is false (1 is not a prime factor; the prime factorisation of 5 is just 5). This is the official answer.

Board questions: 2 and 3 marks ★

CBSE 2023 · 30/2/I (a) Find by prime factorisation the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers. [3]

Ans.

$$18180 = 2^2 \times 3^2 \times 5 \times 101, 7575 = 3 \times 5^2 \times 101$$

$$\text{LCM} = 2^2 \times 3^2 \times 5^2 \times 101 = 90900$$

$$\text{HCF} = \frac{18180 \times 7575}{90900} = 3 \times 5 \times 101 = 1515$$

CBSE 2023 · 30/6/I Find the HCF and LCM of 26, 65 and 117 using prime factorisation. [2]

Ans.

$$26 = 2 \times 13, 65 = 5 \times 13, 117 = 3^2 \times 13$$

$$\text{HCF} = 13, \text{LCM} = 2 \times 3^2 \times 5 \times 13 = 1170$$

CBSE 2023 · 30/6/I Find the HCF and LCM of 72 and 120. [2]

$$\text{Ans. } 72 = 2^3 \times 3^2, 120 = 2^3 \times 3 \times 5 \Rightarrow \text{HCF} = 2^3 \times 3 = 24, \text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

CBSE 2025 · 30/3/I (a) Find the smallest number which is divisible by both 644 and 462. [2]

$$\text{Ans. Smallest common multiple} = \text{LCM. } 644 = 2^2 \cdot 7 \cdot 23, 462 = 2 \cdot 3 \cdot 7 \cdot 11 \Rightarrow \text{LCM} = 2^2 \cdot 3 \cdot 7 \cdot 11 \cdot 23 = 21252$$

CBSE 2024 · 30/2/I Can the number $(15)^n$, n being a natural number, end with the digit 0? Give reasons. [2]

Ans. $15^n = 3^n \times 5^n$. To end with 0 it must have 2 as a prime factor, but by the uniqueness of prime factorisation (FTA) it has only the primes 3 and 5. So **no**, 15^n can never end with 0.

CBSE 2025 · 30/4/I If $p = x^2y^3$, $q = xy^4$, $r = x^5y^2$ (x, y prime), find HCF and LCM of p, q, r and check whether $\text{HCF} \times \text{LCM} = pqr$. [3]

$$\text{Ans. HCF} = xy^2, \text{LCM} = x^5y^4, \text{HCF} \times \text{LCM} = x^6y^6, \text{while } pqr = x^8y^9, \text{so } \text{HCF} \times \text{LCM} \neq pqr.$$

CBSE 2024 · 30/4/I Prove that $\frac{2 - \sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. [3]

Ans.

$$\text{Assume } \frac{2 - \sqrt{3}}{5} = \frac{a}{b} \text{ (a, b integers, } b \neq 0). \quad \frac{1}{2}$$

$$\text{Then } 2 - \sqrt{3} = \frac{5a}{b} \Rightarrow \sqrt{3} = 2 - \frac{5a}{b} = \frac{2b - 5a}{b}.$$

$$\text{RHS is rational but } \sqrt{3} \text{ is irrational: contradiction. So } \frac{2 - \sqrt{3}}{5} \text{ is irrational.} \quad \frac{1}{2}$$

CBSE 2025 · 30/3/I Prove that $\left(5\sqrt{3} + \frac{2}{3}\right)$ is an irrational number, given that $\sqrt{3}$ is an irrational number. [3]

$$\text{Ans. Assume } 5\sqrt{3} + \frac{2}{3} = \frac{a}{b}. \text{ Then } 5\sqrt{3} = \frac{3a - 2b}{3b} \Rightarrow \sqrt{3} = \frac{3a - 2b}{15b}, \text{ which is rational: contradiction.}$$

Hence the number is irrational.

Board questions: proofs ★★

CBSE 2024 · 30/5/I (b) Prove that $(\sqrt{2} + \sqrt{3})^2$ is an irrational number, given that $\sqrt{6}$ is an irrational number. [3]

Ans.

$$(\sqrt{2} + \sqrt{3})^2 = 2 + 3 + 2\sqrt{6} = 5 + 2\sqrt{6}. \quad ①$$

Assume $5 + 2\sqrt{6} = \frac{a}{b}$ (a, b integers, $b \neq 0$). Then $\sqrt{6} = \frac{a - 5b}{2b}$, which is rational. ①

This contradicts that $\sqrt{6}$ is irrational. Hence $(\sqrt{2} + \sqrt{3})^2$ is irrational. ①

CBSE 2023 · 30/2/I Prove that $2 + \sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. [2]

Ans. Assume $2 + \sqrt{3} = \frac{a}{b} \Rightarrow \sqrt{3} = \frac{a - 2b}{b}$, rational: contradiction. So $2 + \sqrt{3}$ is irrational.

CBSE 2025 · 30/I/I Prove that $\frac{1}{\sqrt{5}}$ is an irrational number. [3]

Ans.

Step 1: prove $\sqrt{5}$ is irrational (Template 1, p. 6). ②

Step 2: assume $\frac{1}{\sqrt{5}} = \frac{a}{b}$ ($a, b \neq 0$). Then $\sqrt{5} = \frac{b}{a}$, which is rational: contradiction. Hence $\frac{1}{\sqrt{5}}$ is irrational. ①

CBSE 2023 · 30/5/I 2024 · 30/5/I 2025 · 30/2/I Prove that $\sqrt{3}$ is an irrational number. [3]

Ans.

Assume $\sqrt{3} = \frac{p}{q}$, p, q co-prime integers, $q \neq 0$. ½

Then $p^2 = 3q^2 \Rightarrow 3$ divides $p^2 \Rightarrow 3$ divides $p \dots$ (i). Let $p = 3m$. ①

$9m^2 = 3q^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3$ divides $q^2 \Rightarrow 3$ divides $q \dots$ (ii). ①

(i) and (ii) contradict that p, q are co-prime. So $\sqrt{3}$ is irrational. ½

What the board asked from this chapter (2023–2025 papers I checked)

Topic	2023	2024	2025	Usual marks
Prove $\sqrt{p}$ irrational	4 papers	2 papers	4 papers	3
$a \pm b\sqrt{p} / \frac{1}{\sqrt{p}}$ irrational	1	3	2	2-3
HCF/LCM direct (prime factorisation)	4	1	2	2-3
HCF $\times$ LCM = product (find missing)	—	4	2	1
Remainder / ratio / rooms word problem	2	2	2	1-3
Can a^n end in 0?	2	1	—	1-2

★ **Exam tip:** Pattern: a 1-mark MCQ on HCF/LCM plus a 2- or 3-mark proof. That is 4 easy marks every year.

Case-based questions (4 marks each) Exam-style

Case 1: The school sports day

For the sports day march-past, a school has **312 students in Class IX** and **260 students in Class X**. They must march in rows such that every row has the **same number** of students, and each row has students of only **one** class. On the track, two drummers beat their drums every **12 seconds** and **18 seconds** respectively.

1. What is the maximum number of students in each row? [1]
2. Find the minimum total number of rows. [1]
3. (a) If both drummers beat together at the start, after how many seconds will they next beat together? **OR** (b) How many times will they beat together in 3 minutes (excluding the start)? [2]

Ans.

(i) $312 = 2^3 \times 3 \times 13$, $260 = 2^2 \times 5 \times 13 \Rightarrow \text{HCF} = 2^2 \times 13 = \mathbf{52}$ students per row.

(ii) Rows = $\frac{312}{52} + \frac{260}{52} = 6 + 5 = \mathbf{11}$.

(iii)(a) $\text{LCM}(12, 18) = 2^2 \times 3^2 = \mathbf{36 \text{ seconds}}$. (b) $3 \text{ min} = 180 \text{ s}$; $180 \div 36 = \mathbf{5 \text{ times}}$.

Board case study CBSE 2024 · 30/3/11

Teaching Mathematics through activities is a powerful approach that enhances students' understanding and engagement. Keeping this in mind, Ms. Mukta planned a prime number game for class 5 students. She announces the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to second student. Second student also multiplied it by a prime number and passed it to third student. In this way by multiplying to a prime number, the last student got 173250.

1. What is the least prime number used by students? [1]
2. (a) How many students are in the class? **OR** (b) What is the highest prime number used by students? [2]
3. Which prime number has been used maximum times? [1]

Ans.

$173250 = 2 \times 3^2 \times 5^3 \times 7 \times 11$. The first 2 was announced by the teacher, so the students multiplied by 3, 3, 5, 5, 5, 7, 11. ①

(i) Least prime used by students = **3** (trap: 2 was announced by the teacher, not used by a student). ①

(ii)(a) Number of students = number of primes multiplied = $2 + 3 + 1 + 1 = \mathbf{7}$. (b) Highest prime used = **11**.

(iii) **5** (used 3 times). ①

Case 3: The festival lights Exam-style

Three strings of Diwali lights blink every **48 s**, **72 s** and **108 s**. They are switched on together at 6:00 p.m.

1. Write 48, 72 and 108 as products of primes. [1]
2. At what time will all three blink together again? [1]
3. (a) How many times will they blink together between 6:00 p.m. and 7:00 p.m. (excluding 6:00 p.m.)? **OR** (b) Find $\text{HCF}(48, 72, 108)$ and show that $\text{HCF} \times \text{LCM} \neq 48 \times 72 \times 108$. [2]

Ans.

(i) $48 = 2^4 \cdot 3$, $72 = 2^3 \cdot 3^2$, $108 = 2^2 \cdot 3^3$. (ii) $\text{LCM} = 2^4 \cdot 3^3 = 432 \text{ s} = 7 \text{ min } 12 \text{ s} \Rightarrow \mathbf{6:07:12 \text{ p.m.}}$

(iii)(a) $3600 \div 432 = 8.33 \Rightarrow \mathbf{8 \text{ times}}$. (b) $\text{HCF} = 2^2 \cdot 3 = 12$; $12 \times 432 = 5184$, but $48 \times 72 \times 108 = 373248$, so they are not equal.

20 MCQs (1 mark each) *Exam-style*

1. The prime factorisation of 156 is
(a) $2 \cdot 3 \cdot 13$ (b) $2^2 \cdot 3 \cdot 13$ (c) $2^2 \cdot 39$ (d) $4 \cdot 3 \cdot 13$
2. HCF of 12 and 18 is
(a) 2 (b) 3 (c) 6 (d) 36
3. LCM of 15 and 20 is
(a) 5 (b) 30 (c) 60 (d) 300
4. If $\text{HCF}(a, b) = 4$ and $ab = 96$, then LCM is
(a) 24 (b) 48 (c) 384 (d) 12
5. Which number can end with the digit 0 for some n ?
(a) 4^n (b) 6^n (c) 15^n (d) 20^n
6. The HCF of two co-prime numbers is
(a) 0 (b) 1 (c) their product (d) the smaller one
7. Which of these is irrational?
(a) $\sqrt{16}$ (b) $\sqrt{0.09}$ (c) $\sqrt{12}$ (d) $\frac{22}{7}$
8. If $a = 2^3 \cdot 3$ and $b = 2 \cdot 3^2 \cdot 5$, $\text{HCF}(a, b)$ is
(a) 6 (b) 12 (c) 30 (d) 360
9. For the a, b of Q8, $\text{LCM}(a, b)$ is
(a) 180 (b) 360 (c) 720 (d) 120
10. $\sqrt{2} \times \sqrt{8}$ is
(a) irrational (b) 4 (c) $2\sqrt{2}$ (d) $\sqrt{10}$
11. The least number divisible by 2, 3, 4, 5 and 6 is
(a) 30 (b) 60 (c) 120 (d) 720
12. If 3 divides p^2 (p an integer), then
(a) 9 divides p (b) 3 divides p (c) p is prime (d) $p = 3$
13. $2 + \sqrt{5}$ is
(a) rational (b) irrational (c) an integer (d) a natural number
14. $\text{HCF} \times \text{LCM}$ of 25 and 40 is
(a) 200 (b) 1000 (c) 40 (d) 5
15. The largest number dividing 70 and 125 leaving remainders 5 and 8 is
(a) 13 (b) 65 (c) 875 (d) 1750
16. $7 \times 11 \times 13 + 13$ is
(a) prime (b) composite (c) neither (d) irrational
17. 12^n ends with the digit 0 for
(a) $n = 1$ (b) $n = 5$ (c) every even n (d) no natural n
18. The product of a non-zero rational and an irrational number is
(a) always rational (b) always irrational (c) an integer (d) zero
19. Two numbers are $2x$ and $3x$ with LCM 72. Their HCF is
(a) 6 (b) 12 (c) 24 (d) 36
20. The exponent of 2 in the prime factorisation of 144 is
(a) 2 (b) 3 (c) 4 (d) 6

Answer key

1 (b) · 2 (c) · 3 (c) · 4 (a) · 5 (d) · 6 (b) · 7 (c) · 8 (a) · 9 (b) · 10 (b) · 11 (b) · 12 (b) · 13 (b) · 14 (b) · 15 (a) · 16 (b) · 17 (d) · 18 (b) · 19 (b) · 20 (c)

Hints: Q4 $\text{LCM} = 96/4 = 24$. Q14 $\text{HCF} \times \text{LCM} = 25 \times 40 = 1000$. Q19 $\text{LCM} = 6x = 72 \Rightarrow x = 12$. Q20 $144 = 2^4 \times 3^2$.

Assertion-Reason (10)

(a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

1. A: $\text{HCF}(18, 24) \times \text{LCM}(18, 24) = 432$. R: 18 and 24 are both even numbers.
2. A: $\sqrt{9}$ is a rational number. R: 9 is a perfect square, so $\sqrt{9} = 3$.
3. A: 6^n can never end with 0. R: A number ending in 0 has both 2 and 5 as prime factors.
4. A: $\text{HCF}(7, 11) = 1$. R: 7 and 11 are both odd numbers.
5. A: $5 + \sqrt{3}$ is irrational. R: The sum of a rational and an irrational number is irrational.
6. A: For 12, 15 and 21, $\text{HCF} \times \text{LCM} \neq 12 \times 15 \times 21$. R: $\text{HCF} \times \text{LCM} = \text{product}$ holds only for two numbers.
7. A: 1 is a prime number. R: A prime number has exactly two distinct factors.
8. A: If $\text{LCM}(a, 18) = 36$ and $\text{HCF}(a, 18) = 2$, then $a = 4$. R: $\text{HCF} \times \text{LCM} = \text{product}$ of two numbers.
9. A: $\sqrt{2} \times \sqrt{2}$ is rational. R: The product of two irrational numbers is always irrational.
10. A: $3825 = 3^2 \times 5^2 \times 17$ is its only prime factorisation. R: The Fundamental Theorem of Arithmetic says the factorisation is unique apart from order.

Key: 1 (b) · 2 (a) · 3 (a) · 4 (b) · 5 (a) · 6 (a) · 7 (d) · 8 (a) · 9 (c) · 10 (a)

Very short answer (1 mark each)

1. Write the HCF of the smallest prime and the smallest composite number. Ans. $\text{HCF}(2, 4) = 2$
2. If p and q are distinct primes, find $\text{LCM}(p, q)$. Ans. pq
3. Express 98 as a product of primes. Ans. 2×7^2
4. Is $0.\overline{3}$ rational or irrational? Ans. Rational ($= 1/3$)
5. Give one irrational number between 2 and 3. Ans. $\sqrt{5}$ (or $\sqrt{7}, \sqrt{8}$)
6. If $\text{HCF}(a, b) = 1$, what is $\text{LCM}(a, b)$? Ans. ab
7. Find the sum of the exponents in $360 = 2^3 \cdot 3^2 \cdot 5$. Ans. 6
8. If $\text{LCM}(26, 169) = 338$, find $\text{HCF}(26, 169)$. Ans. $26 \times 169 \div 338 = 13$

Fill in the blanks

1. Every composite number can be expressed as a product of _____. (primes)
2. $\text{HCF}(a, b) \times \text{LCM}(a, b) = \text{_____}$. ($a \times b$)
3. The HCF of two consecutive natural numbers is _____. (1)
4. The LCM of $2^3 \times 3$ and 2×3^2 is _____. (72)
5. $\sqrt{p}$ is _____ for every prime p . (irrational)
6. The decimal expansion of an irrational number is non-terminating and _____. (non-repeating)

True or false

1. The product of two prime numbers is always odd. (False: $2 \times 3 = 6$)
2. $\pi = \frac{22}{7}$ exactly. (False)
3. Every natural number is a real number. (True)
4. HCF of 17 and 34 is 17. (True)

Short answer practice (2 marks each)

1. Find the HCF and LCM of 404 and 96, and verify $\text{HCF} \times \text{LCM} = \text{product}$.

Ans. $404 = 2^2 \cdot 101$, $96 = 2^5 \cdot 3 \Rightarrow \text{HCF} = 4$, $\text{LCM} = 2^5 \cdot 3 \cdot 101 = 9696$. Check: $4 \times 9696 = 38784 = 404 \times 96 \checkmark$

2. Can $(12)^n$ end with the digit 0? Justify.

Ans. $12^n = 2^{2n} \cdot 3^n$ has no factor 5; by FTA it can never end with 0.

3. Prove that $3\sqrt{2}$ is irrational, given that $\sqrt{2}$ is irrational.

Ans. If $3\sqrt{2} = \frac{a}{b}$, then $\sqrt{2} = \frac{a}{3b}$, rational: contradiction. So $3\sqrt{2}$ is irrational.

4. Three bells toll at intervals of 9, 12 and 15 minutes. If they toll together at 8 a.m., when will they next toll together?

Ans. $\text{LCM}(9, 12, 15) = 2^2 \cdot 3^2 \cdot 5 = 180 \text{ min} = 3 \text{ h} \Rightarrow \text{at } 11 \text{ a.m.}$

Short answer practice (3 marks each)

5. Prove that $\sqrt{7}$ is irrational.

Ans. Assume $\sqrt{7} = \frac{p}{q}$ (p, q co-prime). Then $p^2 = 7q^2 \Rightarrow 7 \mid p$; let $p = 7c \Rightarrow q^2 = 7c^2 \Rightarrow 7 \mid q$. So 7 is a common factor: contradiction. Hence $\sqrt{7}$ is irrational.

6. Find the largest number that divides 2053 and 967 and leaves remainders 5 and 7 respectively.

Ans. $\text{HCF}(2048, 960)$: $2048 = 2^{11}$, $960 = 2^6 \cdot 3 \cdot 5 \Rightarrow \text{HCF} = 2^6 = 64$.

7. A shopkeeper has 120 litres of petrol, 180 litres of diesel and 240 litres of kerosene. He wants to sell them in cans of equal size, each can holding only one fuel. Find the greatest capacity of a can and the least number of cans needed.

Ans. $\text{HCF}(120, 180, 240) = 60 \text{ litres}$. Cans = $2 + 3 + 4 = 9$.

8. Prove that $4 - 5\sqrt{2}$ is irrational, given that $\sqrt{2}$ is irrational.

Ans. If $4 - 5\sqrt{2} = \frac{a}{b}$, then $\sqrt{2} = \frac{4b - a}{5b}$, rational: contradiction. So $4 - 5\sqrt{2}$ is irrational.

Long answer (5 marks)

9. (i) Prove that $\sqrt{3}$ is irrational. (ii) Hence show that $2\sqrt{3} - 1$ is irrational. (iii) Is $(2\sqrt{3} - 1)(2\sqrt{3} + 1)$ rational? Explain.

Ans. (i) Template 1 with 3 (p. 12). (ii) If $2\sqrt{3} - 1 = \frac{a}{b}$, then $\sqrt{3} = \frac{a+b}{2b}$, rational: contradiction. (iii) $(2\sqrt{3})^2 - 1^2 = 12 - 1 = 11$: **rational**, even though both factors are irrational.

My mistakes page X → ✓

✗ Common mistake	✓ Correct
HCF uses the greatest powers	HCF: smallest powers of common primes only
HCF × LCM = product for 3 numbers	Only for two numbers
"1 is the smallest prime"	2 is the smallest prime; 1 is neither prime nor composite
Proof without "p, q co-prime"	Write it in the first line; the contradiction depends on it
$\sqrt{3} = \frac{a-2b}{b}$ "is irrational"	The right side is rational, the left side $\sqrt{3}$ is irrational: that is the contradiction
Remainder questions: HCF of the original numbers	Subtract the remainders first: HCF ($a - r_1, b - r_2$)
"6 ⁿ ends in 6, so not 0" (no reason)	Use prime factors and the FTA
Writing $2 \times 2 \times 2 \times 3$ as the final answer	Index form: $2^3 \times 3$

My own mistakes (fill after every test)

Question / what I did	What I should do

Last-night summary

- **FTA**: every composite number = product of primes, unique except for order.
- **HCF** = smallest powers of common primes; **LCM** = greatest powers of all primes. $\text{HCF} \times \text{LCM} = ab$ (two numbers only).
- **Word problems**: "divides / maximum in each group / minimum rooms" ⇒ HCF; "divisible by / together again" ⇒ LCM. Remainders: subtract first.
- **Ends in 0** needs prime factors 2 and 5: $4^n, 6^n, 12^n, 15^n$ never; $10^n, 20^n$ always.
- $\sqrt{p}$ **irrational**: assume $\frac{p}{q}$ co-prime → square → p divisible → q divisible → contradiction.
- $a + b\sqrt{p}$: assume rational, isolate $\sqrt{p}$ = rational, contradiction.
- rational ± irrational = irrational; non-zero rational × irrational = irrational; irrational × irrational may be rational.

★ **Exam tip**: Board pattern: 1 MCQ (HCF/LCM) + 1 proof (2–3 marks) every year. Revise p. 3, 4 and 6 the night before.